ECE 374 B - Fall 2024

Homework 2

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/fa24/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com/fa24) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. Let L be the set of all strings in $\{0,1\}^*$ that contain exactly two occurrences of the substring 001.

   (a) Describe a DFA that over the alphabet $\Sigma = \{0,1\}$ that accepts the language L. Argue that your machine accepts every string in L and nothing else, by explaining what each state in your DFA means. (You may either draw the DFA or describe it formally, but the states Q, the start state s, the accepting states A, and the transition function δ must be clearly specified.)

   (b) Give a regular expression for L, and briefly argue that why expression is correct.

2. In certain programming languages, comments appear between delimiters such as $/\#$ and $\#/$. Let C be the language of all valid delimited comment strings. A member of C must begin with $/\#$ and end with $\#/$ but have no intervening $\#/$. For simplicity, assume that the alphabet for C is $\Sigma = \{a, b, /, \#\}$.

   (a) Give a DFA that recognizes C.

   (b) Give a regular expression that generates C.

3. For each of the following languages, draw (or describe formally) an NFA that accepts them. Your automata should have a small number of states. Provide a short explanation of your solution, if needed.

   (a) All strings over $\{a, b, c\}^*$ in which every nonempty maximal substring of consecutive a's is of even length.

   (b) $\Sigma^* a \Sigma^* b \Sigma^* c \Sigma^*$

   (c) All strings in $w \in a^*$ of length that is divisible by at lease one of the following numbers 2, 3, 5. For full credit your automata should have less than (say) 15 states.

   (d) All strings in $w \in a^*$ of length that is **NOT** divisible by at lease one of the following numbers 2, 3, 5.

4. (a) For any string $w = w_1 w_2 \dots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \dots w_2 w_1$. For any language L, let $L^R = \{w^R \mid w \in L\}$. Show that if L is regular, so is L^R.

(b) Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

Σ_3 contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows of 0s and 1s. Consider each row to be a binary number and let

$$B = \{w \in \Sigma_3^* \mid \text{the bottom row of } w \text{ is the sum of the top 2 rows}\}.$$

For example,

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \notin B.$$

Show that B is regular. (Hint: Working with B^R is easier. Use the result of part (a).)