ECE 374 B - Fall 2024

Homework 9

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/fa24/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com/fa24) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. For any integer k, the problem kSAT is defined as follows:

   - INPUT: A boolean formula Φ in conjunctive normal form, with exactly k distinct literals in each clause.
   - OUTPUT: TRUE if Φ has a satisfying assignment, and FALSE otherwise.

   (a) Describe and analyze a polynomial-time reduction from 2SAT to 3SAT, and prove your reduction is correct.

   (b) Describe and analyze a polynomial-time algorithm for 2SAT. *[Hint: This problem is strongly connected to topics earlier in the semester.]*

   (c) Why don't these results imply a polynomial-time algorithm for 3SAT?

2. Prove the following problems are NP-hard.

   (a) Given an *undirected* graph G, does G contain a simple path that visits all but 17 vertices?

   (b) Given an *undirected* graph G with *weighted* edges, compute a *maximum-diameter* spanning tree of G. (The diameter of a tree T is the length of a longest path in T.)

Let M be a Turing machine, let w be an arbitrary input string, and let s and t be positive integers. We say that M accepts w *in space* s if M accepts w after accessing at most the first s cells on its tape, and M accepts w *in time* t if M accepts w after at most t transitions. Prove that the following languages are decidable or undecidable:

3. (a) $\{\langle M, w\rangle \mid M \text{ accepts } w \text{ in time } |w|^2\}$

   (b) $\{\langle M\rangle \mid M \text{ accepts at least one string } w \text{ in time } |w|^2\}$

   (c) $\{\langle M, w\rangle \mid M \text{ accepts } w \text{ in space } |w|^2\}$

   (d) $\{\langle M\rangle \mid M \text{ accepts at least one string } w \text{ in space } |w|^2\}$

4. Let $(\Sigma = \{0, 1\})$:

$$X = \left\{ \begin{array}{ll} 0w & w \in A_{TM} \\ 1w & w \in \bar{A}_{TM} \end{array} \right\}$$

Show that neither X nor $\bar{X}$ is recursively-enumerable.