This is a review of context-free grammars from the lecture on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \to \epsilon \mid S\,\texttt{(}\,S\,\texttt{)} \qquad \text{properly nested parentheses}$$

  Here is a different grammar for the same language:

$$S \to \epsilon \mid \texttt{(}\,S\,\texttt{)}\mid SS \qquad \text{properly nested parentheses}$$

- $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of $\mathtt{0}$s followed by a different number of $\mathtt{1}$s.

$$\begin{aligned} S &\to A \mid B & \{\mathtt{0}^m\mathtt{1}^n\}\, m \neq n\\ A &\to \mathtt{0}A \mid \mathtt{0}C & \{\mathtt{0}^m\mathtt{1}^n\}\, m > n\\ B &\to B\mathtt{1} \mid C\mathtt{1} & \{\mathtt{0}^m\mathtt{1}^n\}\, m < n\\ C &\to \epsilon \mid \mathtt{0}C\mathtt{1} & \{\mathtt{0}^m\mathtt{1}^n\}\, m = n \end{aligned}$$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

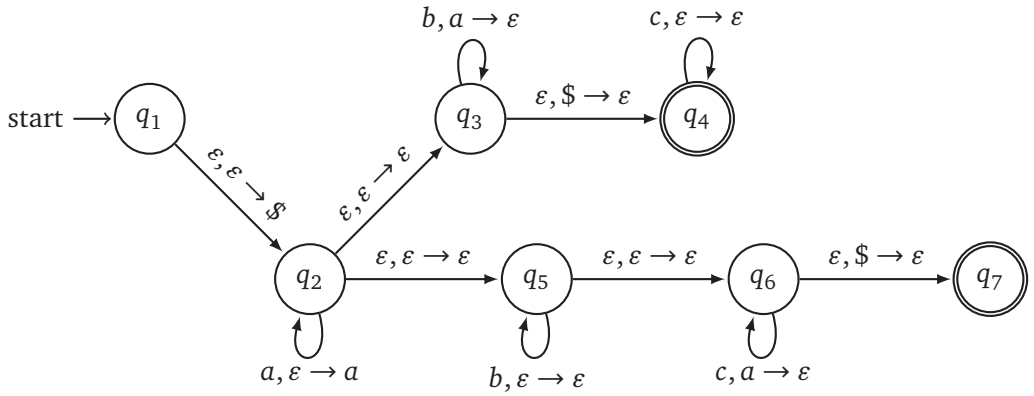
1. $\left\{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\right\}$
2. $\{\mathtt{0}^m\mathtt{1}^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string $\mathtt{01}$.]
3. $\{\mathtt{0},\mathtt{1}\}^* \setminus \left\{\mathtt{0}^{2n}\mathtt{1}^n \mid n \geq 0\right\}$

   [Hint: Extend the previous grammar. What is missing?]

The next few problems deal with push-down automata (PDA). The goal of these problems is to simply gain an understanding of PDAs which are the machines needed to recognize a context-free language:

4. What language does the following push-down automata recognize (Hint: This is a non-deterministic automata as most PDAs are)?



5. Develop the PDA for the language:

$$L = \{w \text{ is a palidrome and } w \in \{\mathtt{0}, \mathtt{1}\}^*\} \tag{1}$$

Work on these later:

4. $\left\{w \in \{\mathtt{0}, \mathtt{1}\}^*\right\} \#(\mathtt{0}, w) = 2 \cdot \#(\mathtt{1}, w)$ – Binary strings where the number of **0**s is exactly twice the number of **1**s.

5. $\{\mathtt{0}, \mathtt{1}\}^* \setminus \{ww\}\, w \in \{\mathtt{0}, \mathtt{1}\}^*$.

   *[Anti-hint: The language $\{ww\}\, w \in \mathtt{0}, \mathtt{1}^*$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*

6. Convert the following CFG into a PDA:

$$S \to \mathtt{a}B\mathtt{c} \mid \mathtt{ab}$$
$$B \to SB \mid \varepsilon$$