In lecture, we described an algorithm of Karatsuba that multiplies two n-digit integers using $O(n^{\lg 3})$ single-digit additions, subtractions, and multiplications. In this lab we'll look at some extensions and applications of this algorithm.

1. Describe an algorithm to compute the product of an n-digit number and an m-digit number, where $m < n$, in $O(m^{\lg 3-1} n)$ time. *Hint:* Break up the bigger number into chunks with m bits each.

2. Describe an algorithm to compute the decimal representation of 2^n in $O(n^{\lg 3})$ time. (The standard algorithm that computes one digit at a time requires $\Theta(n^2)$ time.)

3. Describe a divide-and-conquer algorithm to compute the decimal representation of an arbitrary n-bit binary number in $O(n^{\lg 3})$ time. *[Hint: Let $x = a \cdot 2^{n/2} + b$. Watch out for an extra log factor in the running time.]*

Think about later:

4. Suppose we can multiply two n-digit numbers in $O(M(n))$ time. Describe an algorithm to compute the decimal representation of an arbitrary n-bit binary number in $O(M(n) \log n)$ time.