Equivalence of DFAs, NFAs & Regular Expressions

Sides based on material by Profs. Kani, Erickson, Chekuri, et. al.

All mistakes are my own! - Ivan Abraham (Fall 2024)

Image by ChatGPT (probably collaborated with DALL-E)

Goal of lecture

- The point of this lecture is to establish that we gain no additional computational chops by choosing one of DFA/NFA/ RegEx (Regular Expressions) over the other.

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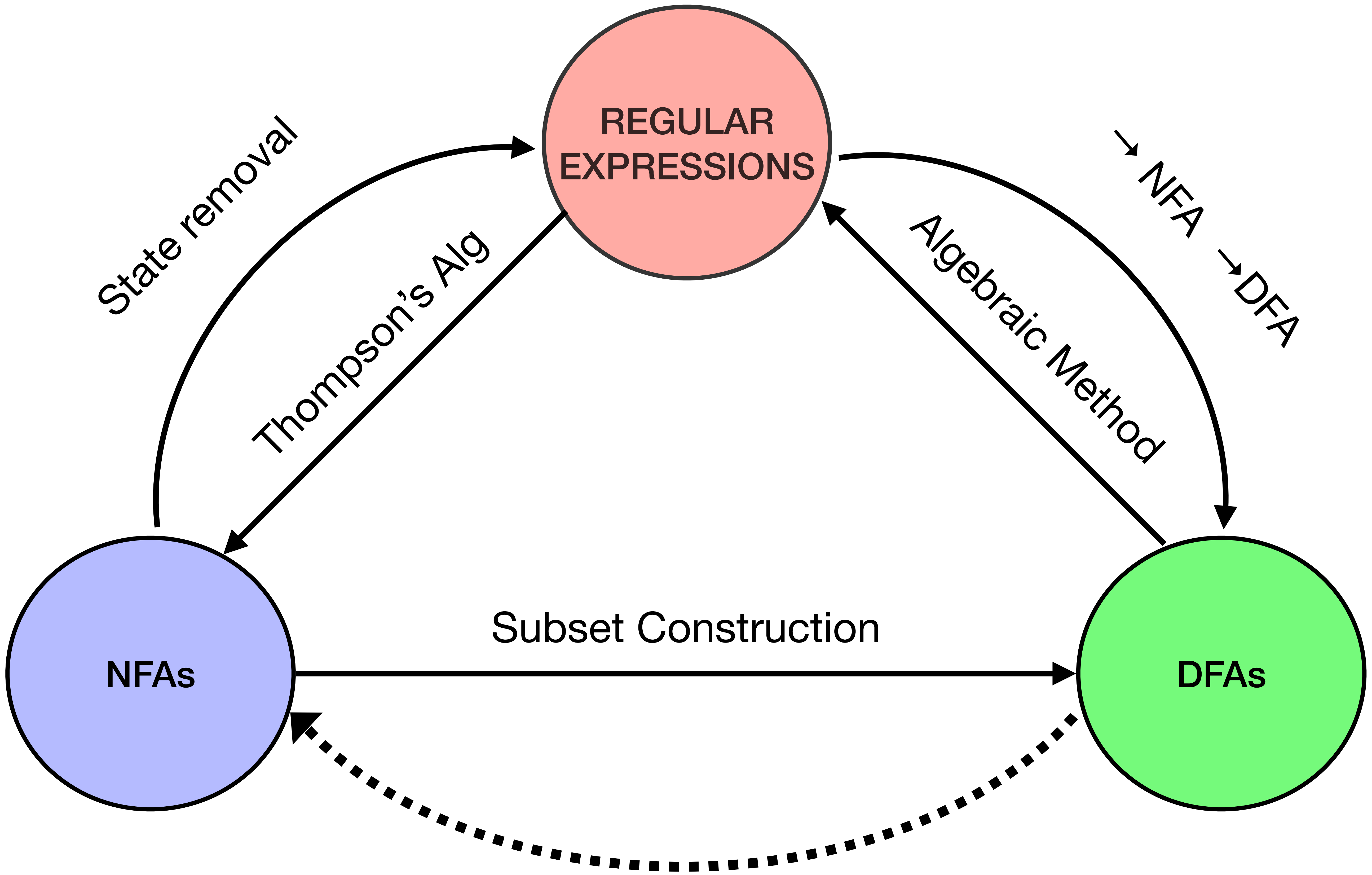
- They all represent the same class of language - *regular languages.*



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Stephen Kleene student of Alonzo Church

→ lambda calculus

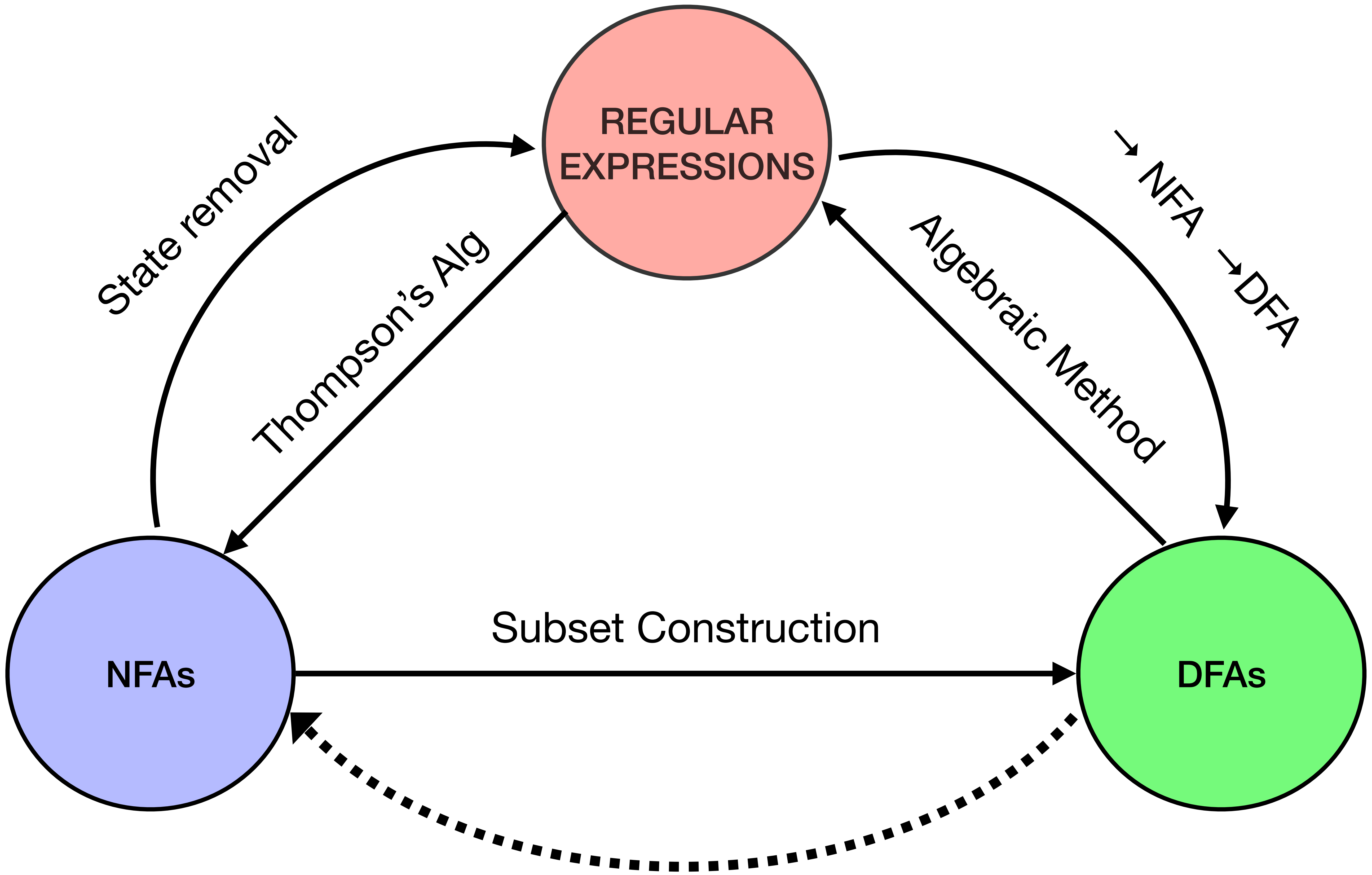
Church-Turing Thesis.

A language L can be described by a regular expression if and only if L is the language accepted by a DFA. $\subseteq$ NFAS $\Longleftrightarrow$ RegEx

Kleene's Theorem ~ 1951

Outline of lecture

- Each of the arrows in the figure on the right could be *formally* proved … but



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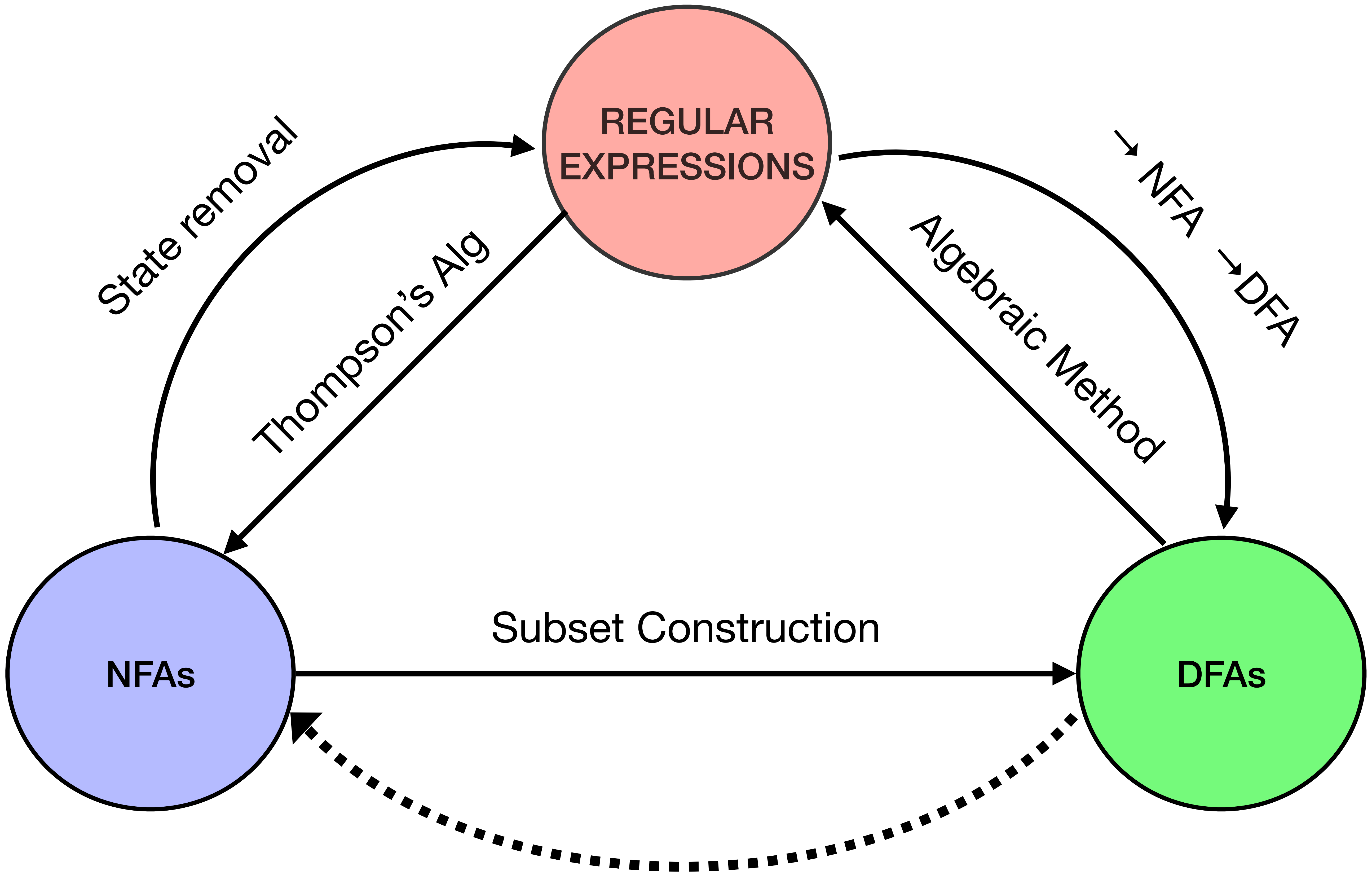
Outline of lecture

- Each of the arrows in the figure on the right could be *formally* proved … but
    - We will only look at the *Subset Construction* formally.



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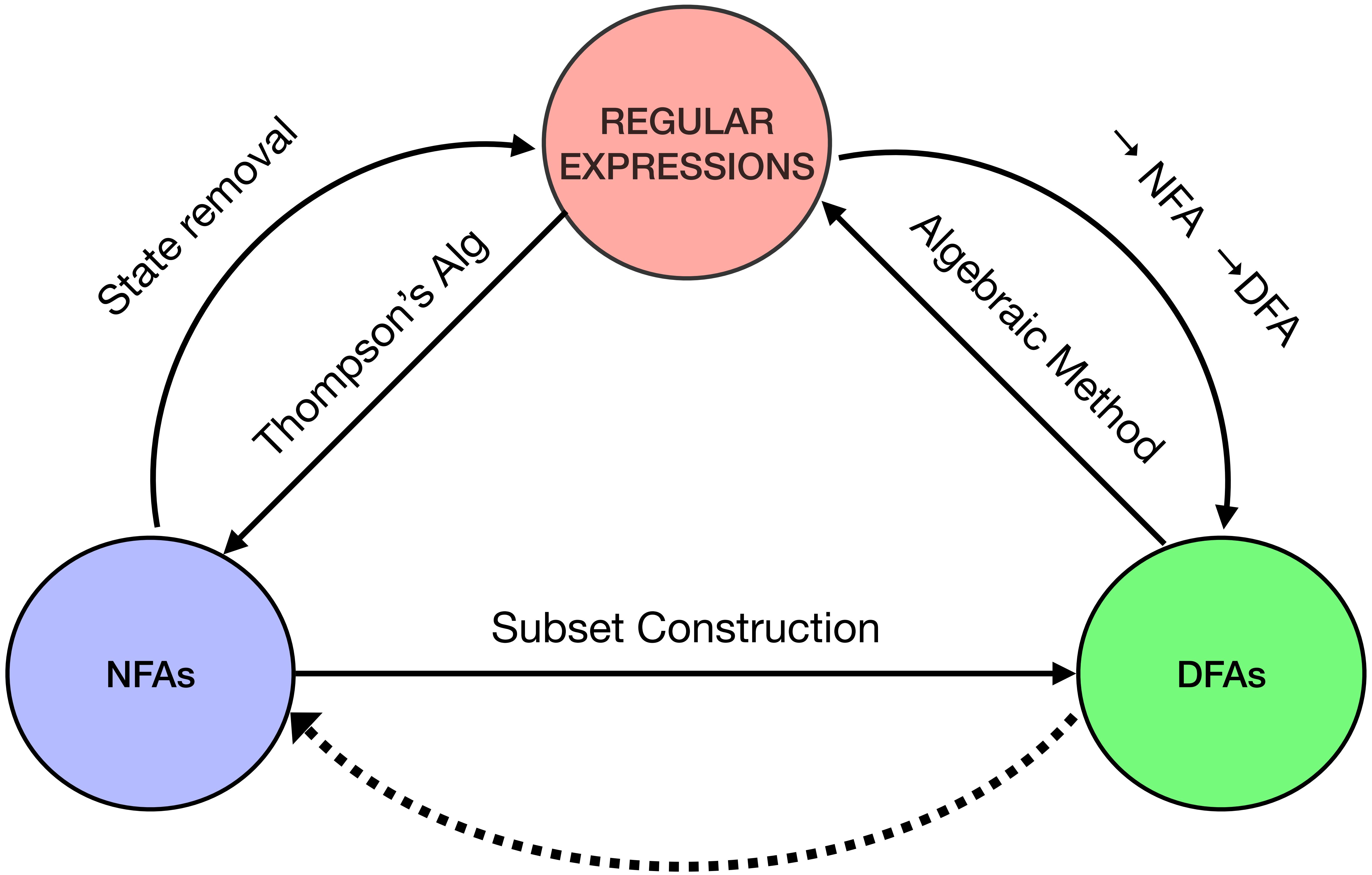
Outline of lecture

- Each of the arrows in the figure on the right could be *formally* proved … but
  - We will only look at the *Subset Construction* formally.
  - For the remaining, we will "prove by example."



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Equivalence of DFAs and NFAs

Formal definitions

Deterministic Finite Automaton

Recall that the formal definition of a DFA is as follows. A DFA is a 5-tuple

$$M = \left(Q, \Sigma, \delta, q_0, F\right)$$

where

Q is a set of finite states.

Σ is a finite set of tokens/characters alphabet

$\delta : Q \times \Sigma \to Q$ is called a transition function

q_0 : start state

F : final / accept states

Formal definitions

Nondeterministic Finite Automaton

Recall that the formal definition of an NFA is as follows. A NFA is a 5-tuple

$$N = \left(Q, \Sigma, \delta, q_0, F\right)$$

where

- Q is a finite set of *states*,
- Σ is a finite set of tokens/characters called the *alphabet*,
- $\delta : Q \times \Sigma \cup \varepsilon \to 2^Q$ is a *transition rule* that encodes state changes when a token from the alphabet is consumed,
- $q_0 \in Q$ is a single distinguished state called the *start state*,
- $F \subseteq Q$ is a set of distinguished states called the *accept* or *final states*.

Equivalence of NFAs and DFAs

Key difference

- NFAs we have introduced allow spontaneous transitions (called ε-transitions)
- NFAs could be in multiple states simultaneously 2^Q or $P(Q)$
- NFAs need not spell out every transition

All DFAs are NFAs!

Take an NFA such that $|\delta(q,\sigma)| \leq 1$

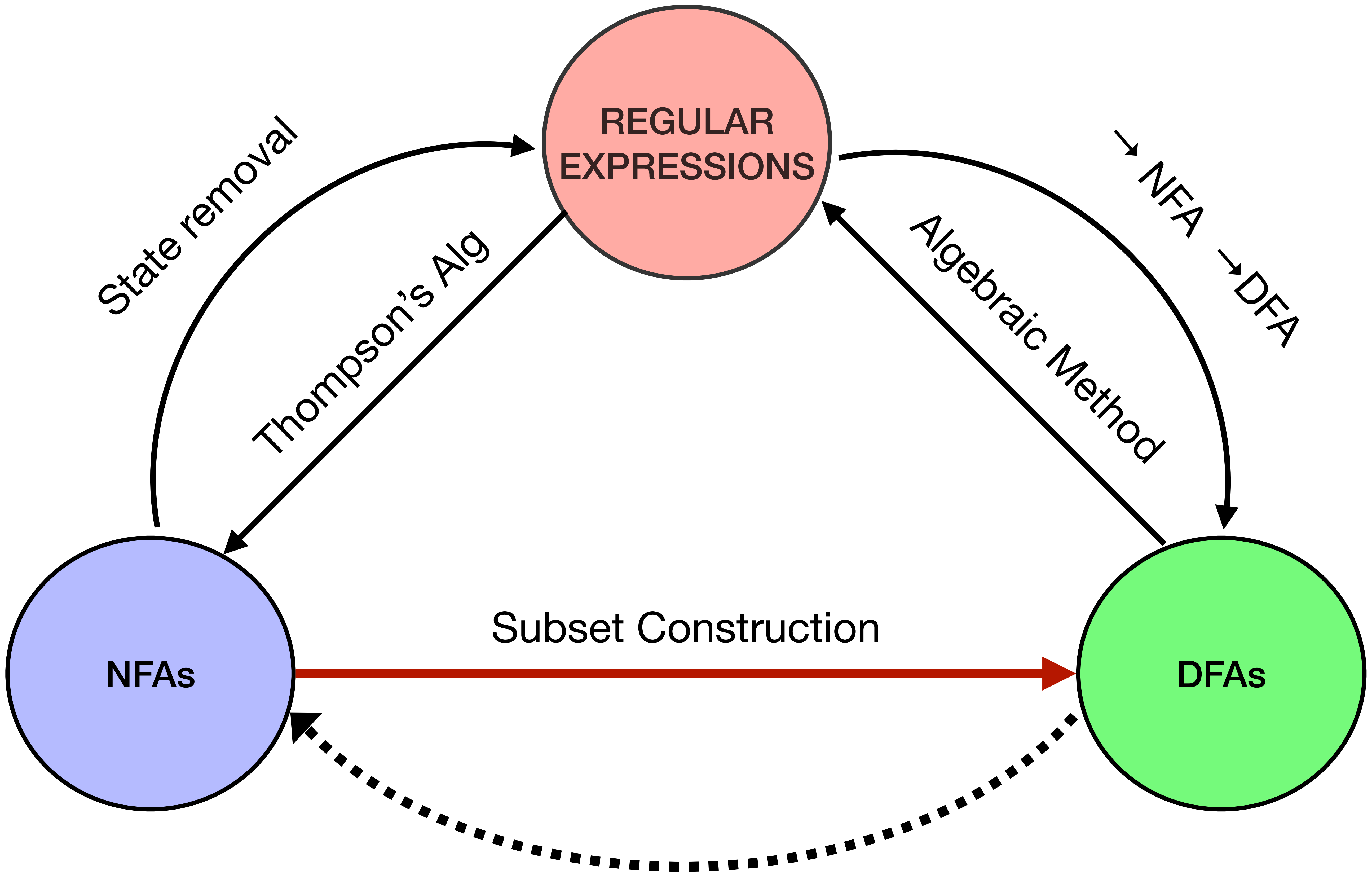
$\delta(q,\sigma) \neq \emptyset$

$\forall q \in Q, \sigma \in \Sigma$

and does not have any ε-transitions. Then such an NFA is A DFA!

Equivalence of NFAs and DFAs

- Thus, we only need to show that for every NFA N, there exists an equivalent DFA M



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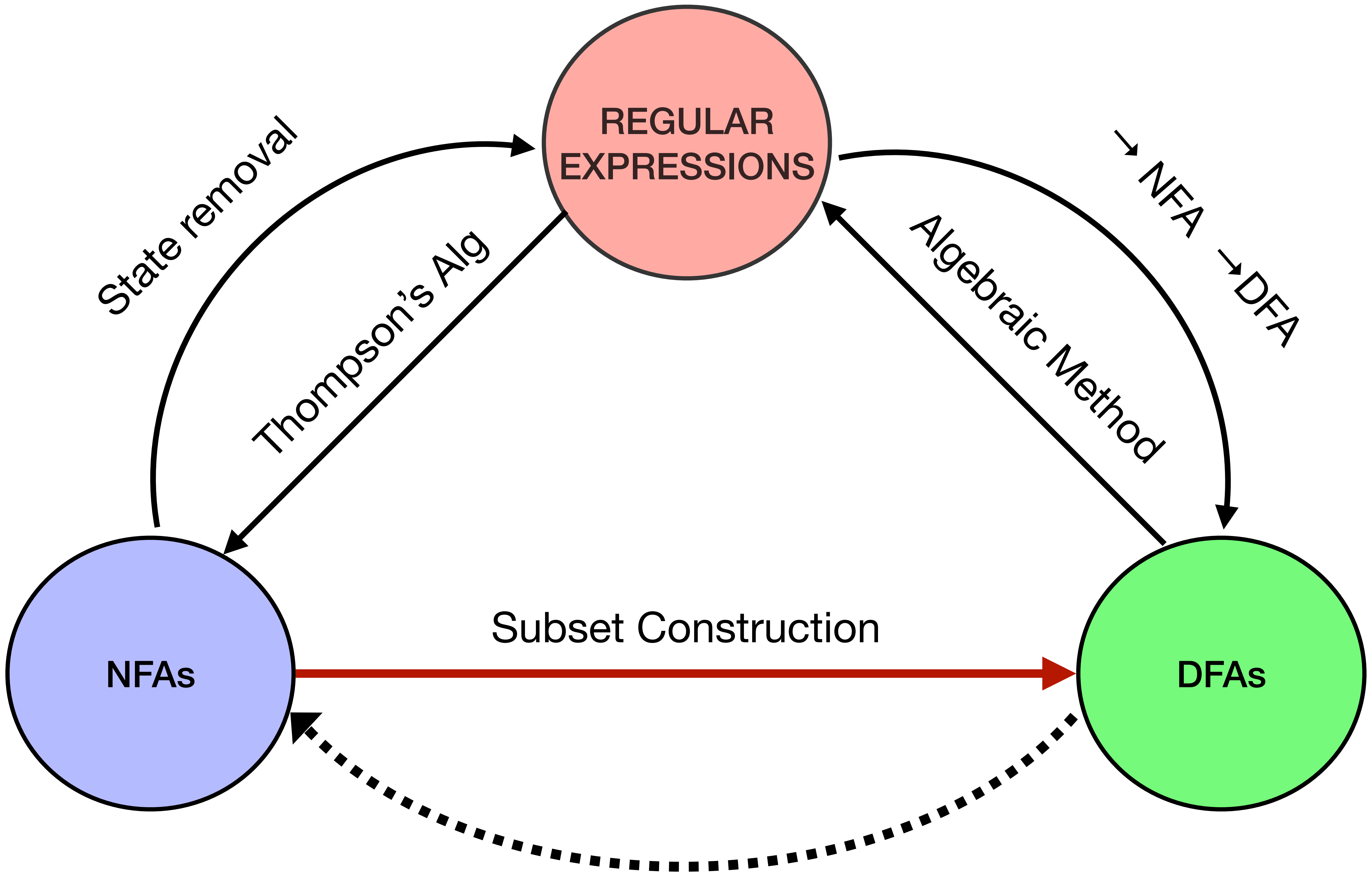
Equivalence of NFAs and DFAs

- Thus, we only need to show that for every NFA N, there exists an equivalent DFA M
  - What does it mean for two finite automata to be *equivalent*?



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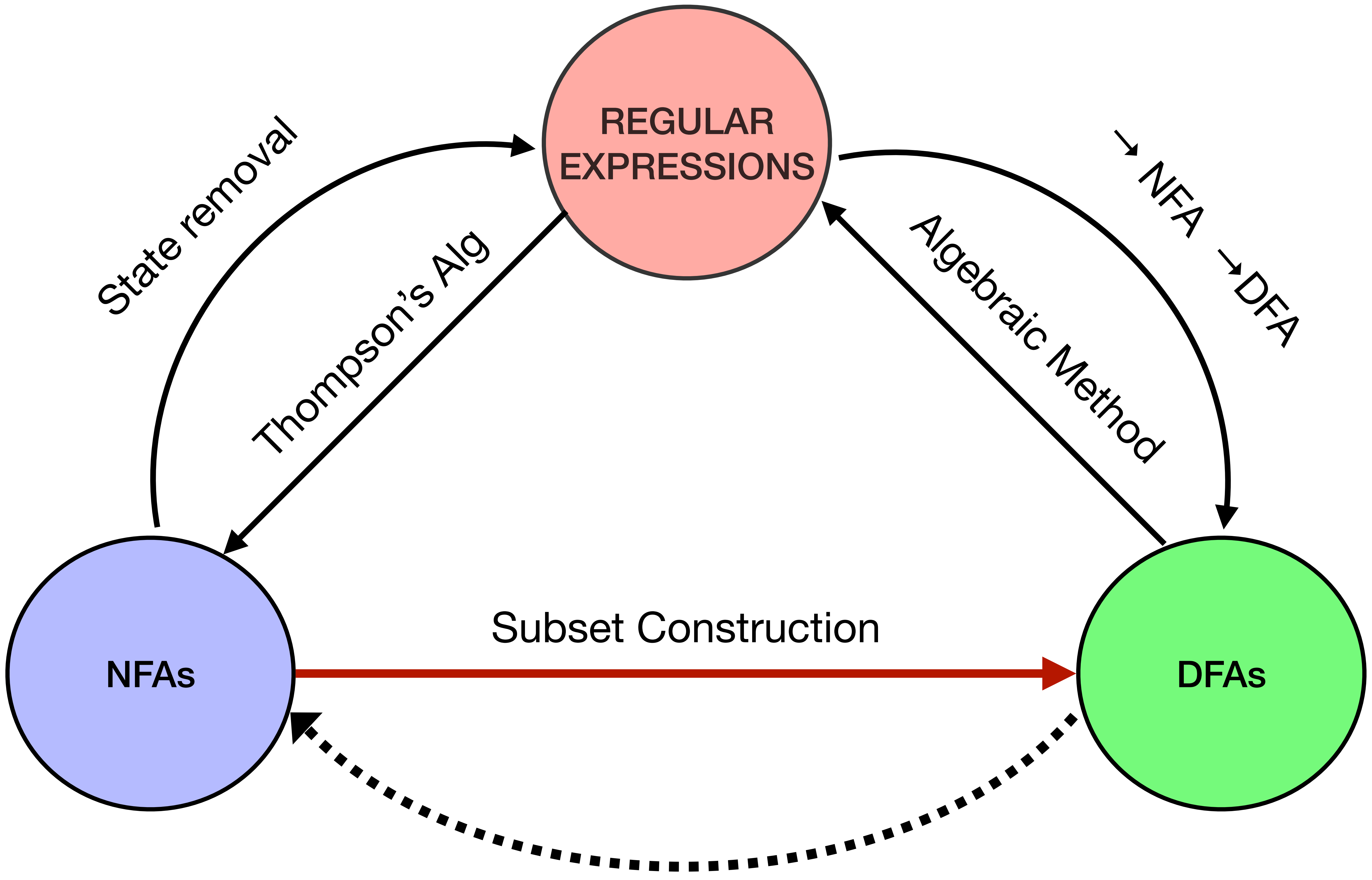
Equivalence of NFAs and DFAs

- Thus, we only need to show that for every NFA N, there exists an equivalent DFA M
  - What does it mean for two finite automata to be *equivalent*?
  - Given N, need to show can construct M such that

$$L(M) = L(N)$$



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Equivalence of NFAs and DFAs

Extended transition functions

notation $\hat{\delta}$ same as δ^*

- For a DFA M we can say M accepts a string w if $\hat{\delta}(q_0, w) \in F$ where $\hat{\delta}_M : Q \times \Sigma^* \to Q$ is the extended transition function defined recursively
  - $\hat{\delta}_M(q, w) = q$ if $w = \varepsilon$
  - $\hat{\delta}_M(q, w) = \hat{\delta}_M(\delta(q, a), x)$ if $w = ax$ for $a \in \Sigma$, $x \in \Sigma^*$

$a \neq \varepsilon$

$a \neq \varepsilon$: $\hat{\delta}_M(q, \varepsilon x) = \hat{\delta}_M(q, x) = \hat{\delta}_M(\delta(q, \varepsilon), x)$

$\delta(q, \varepsilon)$: not defined

$= \hat{\delta}_M(q, x)$

Equivalence of NFAs and DFAs

Extended transition functions

- Define $E(q)$ to be the ε-reach of $q \in Q$. That is, let $E(q)$ be the set of states reachable from q by following zero or more ε arrows.

Equivalence of NFAs and DFAs

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$$E(R) := \bigcup_{r \in R} E(r)$$

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$$\hat{\delta}_N(q, w) = E(q) \quad \text{if } w = \varepsilon$$

$$\hat{\delta}_N(q, w) = \bigcup_{p \in \hat{\delta}_N(q, x)} E(\delta(p, a)) \quad \text{if } w = xa \text{ where } a \in \Sigma$$

↳ could be a set

Note: Definition here is equivalent to definition in Lecture #3. To be really sure you understand try this on Example in pp. 32 of Lec #3.

Equivalence of NFAs and DFAs

Subset construction method

- Now we can say a DFA M and NFA N are equivalent if their extended transitions $\hat{\delta}_M$ and $\hat{\delta}_N$ agree on all words w.

- Given $N = (Q, \Sigma, \delta, q_0, F)$ we should construct a DFA $M = (Q', \Sigma', \delta', q_0', F')$ such that $L(M) = L(N)$

1) → $\Sigma' = \Sigma$

2) $Q' = 2^Q$ or $P\,Q$ → at any instant N will be is a collection of states from Q ⇒ will always be a subset of 2^Q.

Equivalence of NFAs and DFAs

Subset construction method

- Next, we must define the transition rule for M incorporating those ε -transitions of N.

Equivalence of NFAs and DFAs

Subset construction method

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- From any state R in M (which, remember, is a set of states), if we consume a token a, we need to follow any edges labeled a, and then we need to take any ε-transitions from there. Thus we get:

$$\delta'(R, a) := \bigcup_{q \in R} E(\delta(q, a))$$

Equivalence of NFAs and DFAs

Subset construction method

- Finally, it remains to specify the start and accept states q_0' and F' respectively.

$q_0' = E(q_0)$

- Final states F',

$F' = \{ R \in Q' \mid R \cap F \neq \emptyset \}$

Need to include any subset of 2^Q that has a final state of N.

Equivalence of NFAs and DFAs

Subset construction method

- That completes the specification of a DFA M mimicking the functioning of an NFA N.
- Is the proof complete?

Equivalence of NFAs and DFAs

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Equivalence of NFAs and DFAs

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    - One way to finish the proof is to show $\hat{\delta}_N(q_0, w) = \hat{\delta}_M(q_0', w)$ for ***all*** $w \in \Sigma^*$
    - It can be done using induction on $|w|$ and fair bit of definition chasing.

Example - subset construction

We write software to automate tasks …

…. loops, subroutines and functions to avoid repetition and tedium …

… so why reinvent the wheel?

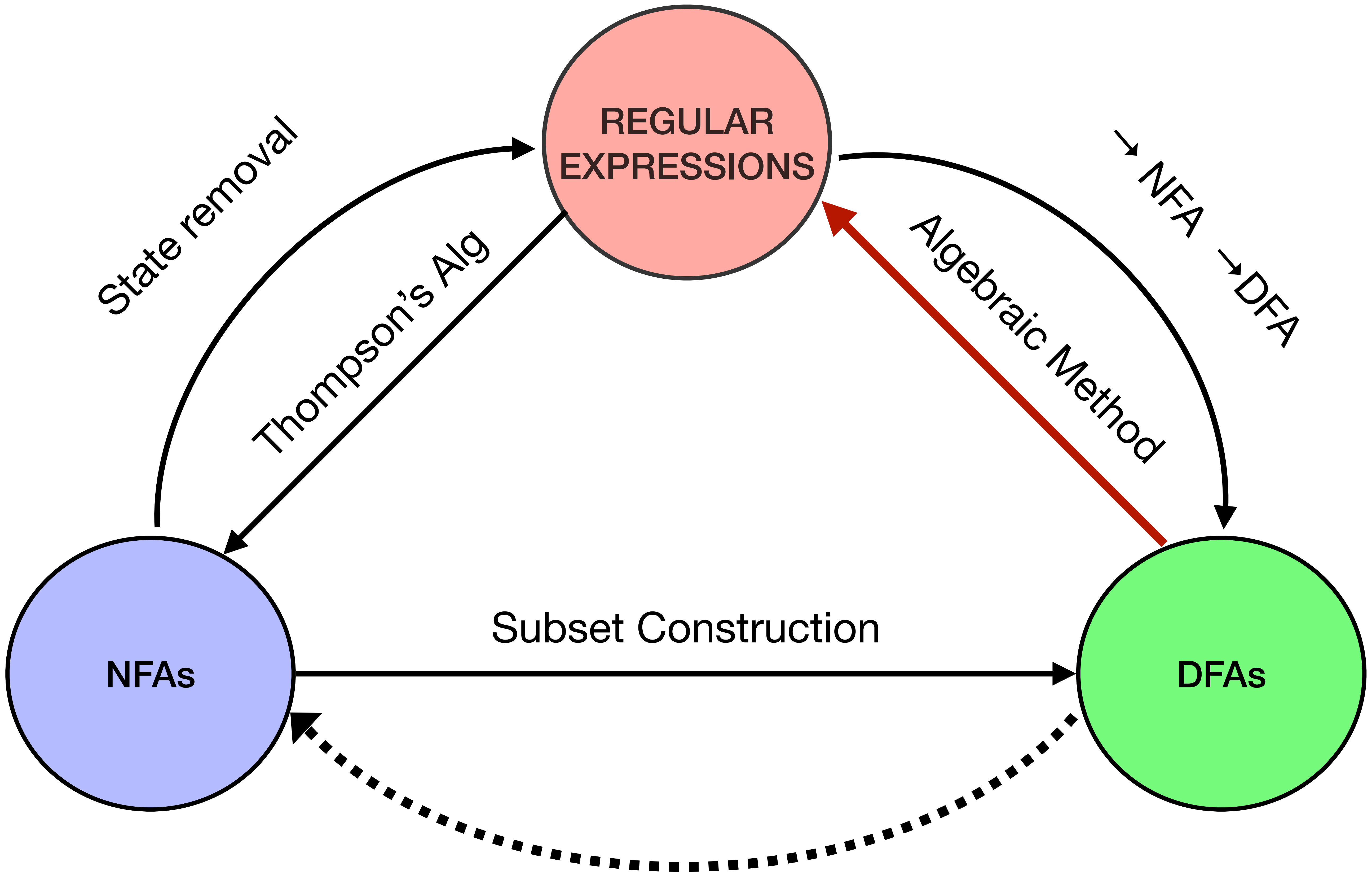
Standford's CS 103 Notes: Guide to the Subset Construction

Equivalence of DFAs and Regular Expressions

Converting a DFA to Regular Expression

Algebraic method

- Next, let us look at how one might construct a **regular expression out of a DFA**:
  - Highlighted red arrow in diagram



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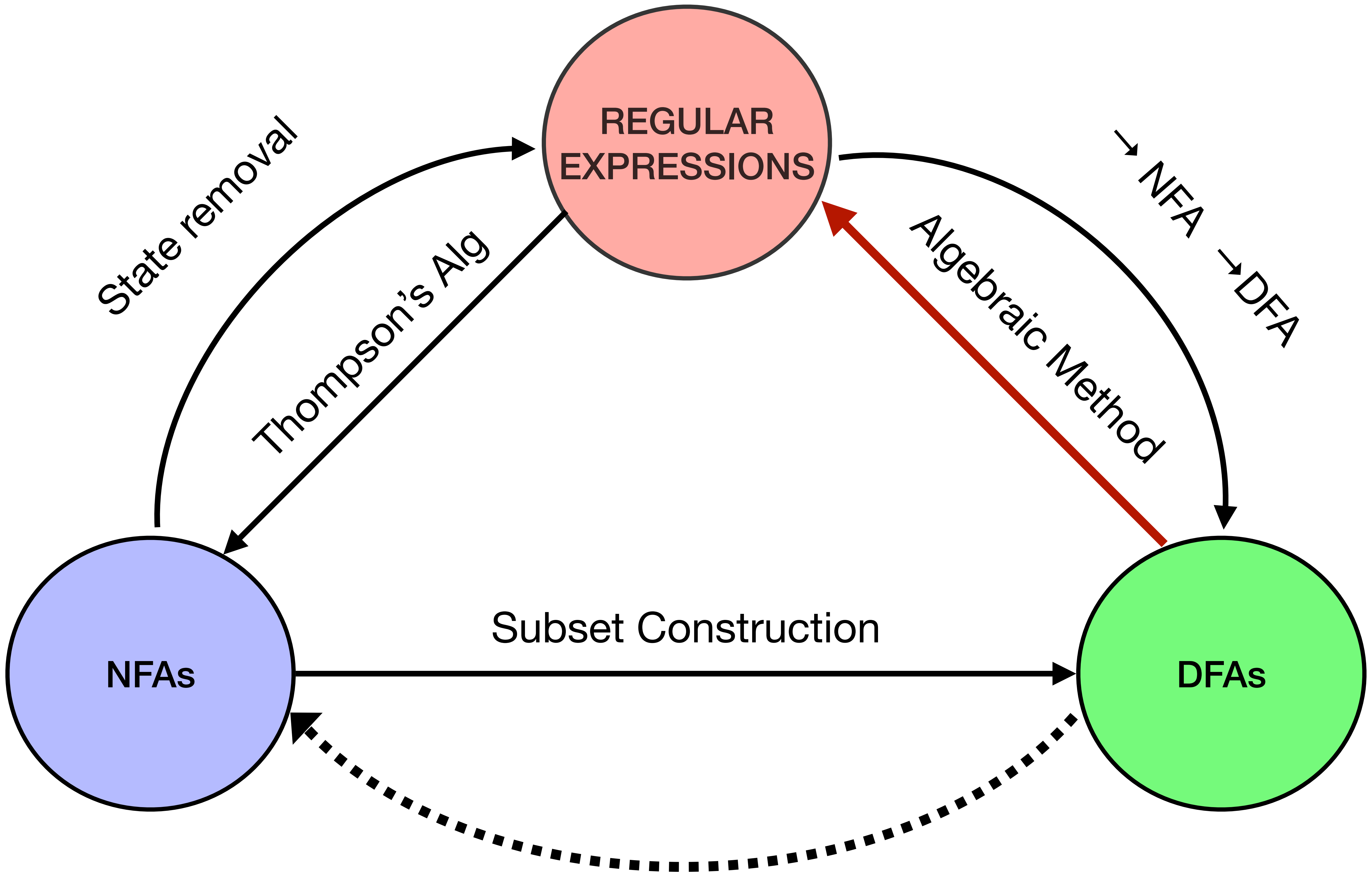
Converting a DFA to Regular Expression

Algebraic method

- Next, let us look at how one might construct a **regular expression out of a DFA**:
  - Highlighted red arrow in diagram
- Called *algebraic* because we end up solving a system of equations



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Converting a DFA to Regular Expression

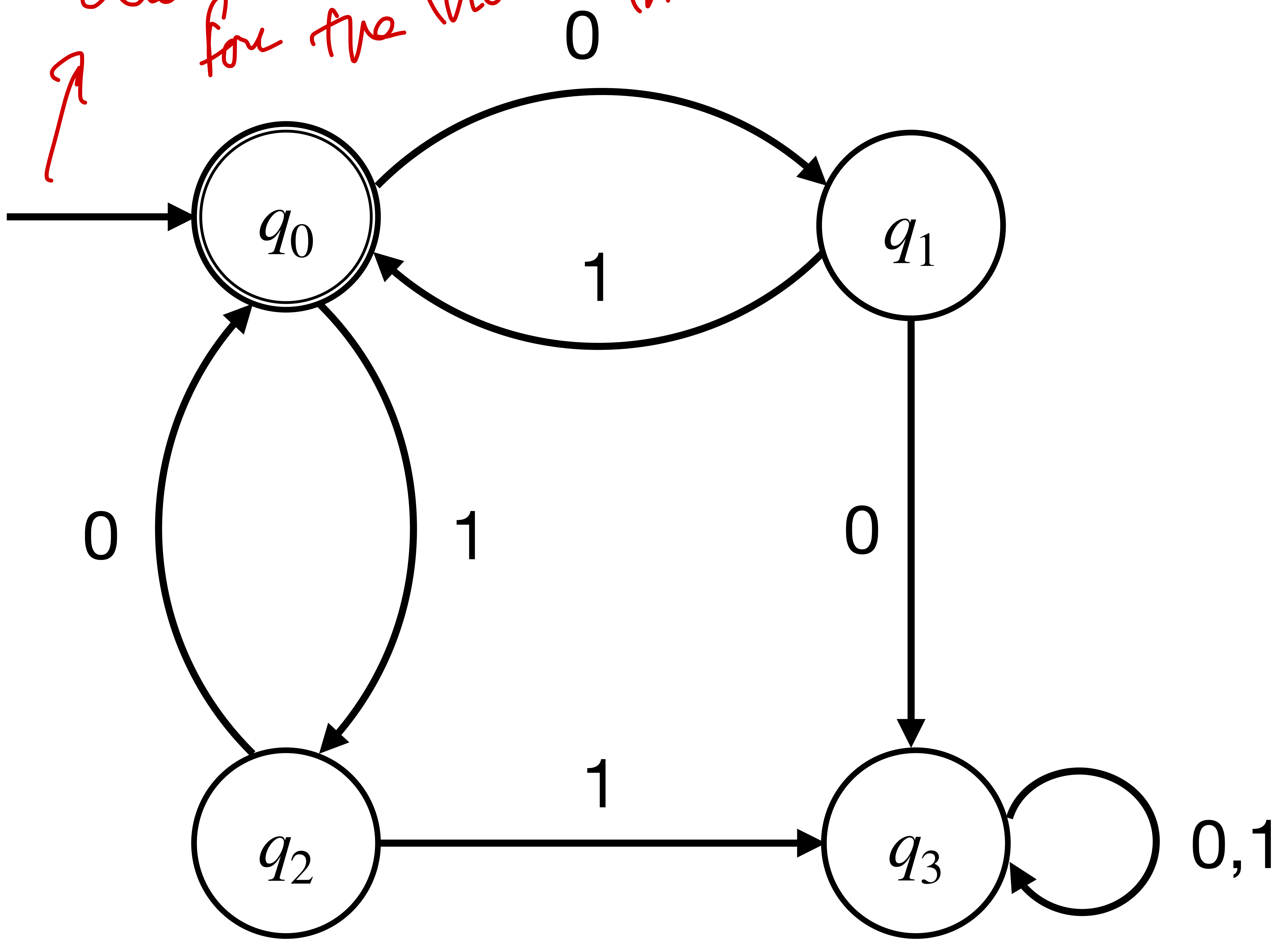
Algebraic method - Example

add an ε for the incoming arrow in start state

Key point: We can write a transition to a state as a juxtaposition of the prior state with the consumed token.

Example

$q_1 = q_0 \cdot 0$



Converting a DFA to Regular Expression

Algebraic method - Example

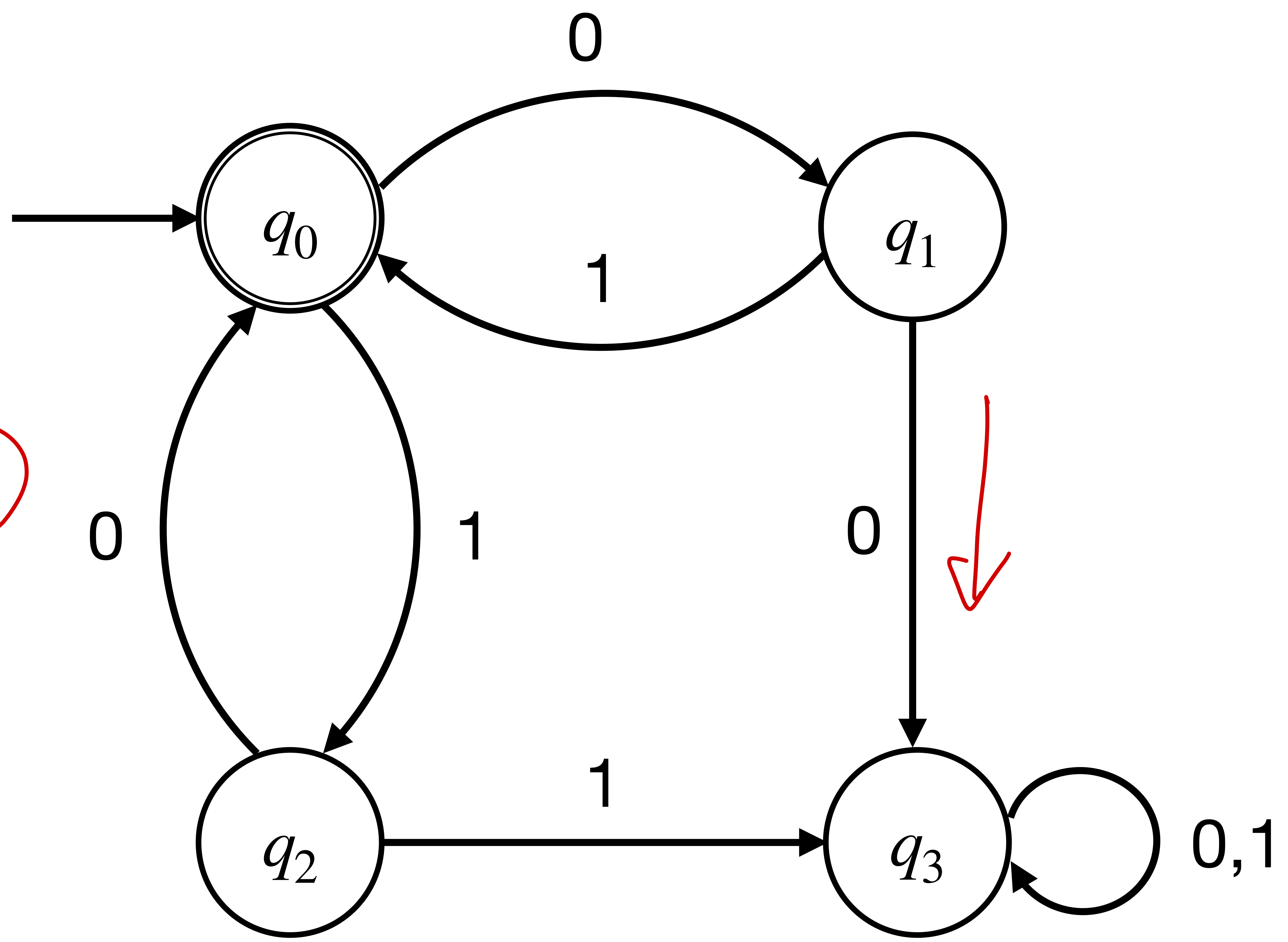
$q_0 = \varepsilon + q_1 \cdot 1 + q_2 \cdot 0$

$q_1 = q_0 \cdot 0$

$q_2 = q_0 \cdot 1$

$q_3 = q_1 \cdot 0 + q_2 \cdot 1 + q_3 \cdot (0 + 1)$

↳ won't use it



Converting a DFA to Regular Expression

Algebraic method - Example

- $q_0 = \epsilon + q_1 1 + q_2 0$
- $q_1 = q_0 0$
- $q_2 = q_0 1$
- $q_3 = q_1 0 + q_2 1 + q_3 (0 + 1)$

- $q_0 = \epsilon + q_1 1 + q_2 0$

$q_0 = \varepsilon + q_0 01 + q_0 10$

$= \varepsilon + q_0 (01 + 10)$

use Arden's Lemma.

$R = Q + RP$

$= QP^*$

Arden's lemma

Proof sketch

- Show that $R = Q + RP = QP^*$

$$R = Q + RP$$

$$= Q + (Q + RP)P$$

$$= Q + [Q + (Q + RP)P]P$$

$$\vdots$$

$$R = Q[\varepsilon + P + P^2 + P^3 + \cdots] = QP^*$$

Converting a DFA to Regular Expression

Algebraic method - Example

- $q_0 = \epsilon + q_1 1 + q_2 0$
- $q_1 = q_0 0$
- $q_2 = q_0 1$
- $q_3 = q_1 0 + q_2 1 + q_3 (0 + 1)$

$$R = Q + RP = QP^*$$

- $q_0 = \epsilon + q_1 1 + q_2 0$
- $q_0 = \epsilon + q_0 01 + q_0 10$
- $q_0 = \epsilon + q_0 (01 + 10)$

R Q R P

Apply **Arden's Lemma**

$q_0 = \varepsilon . (01+10)^*$

$= (01+10)^*$

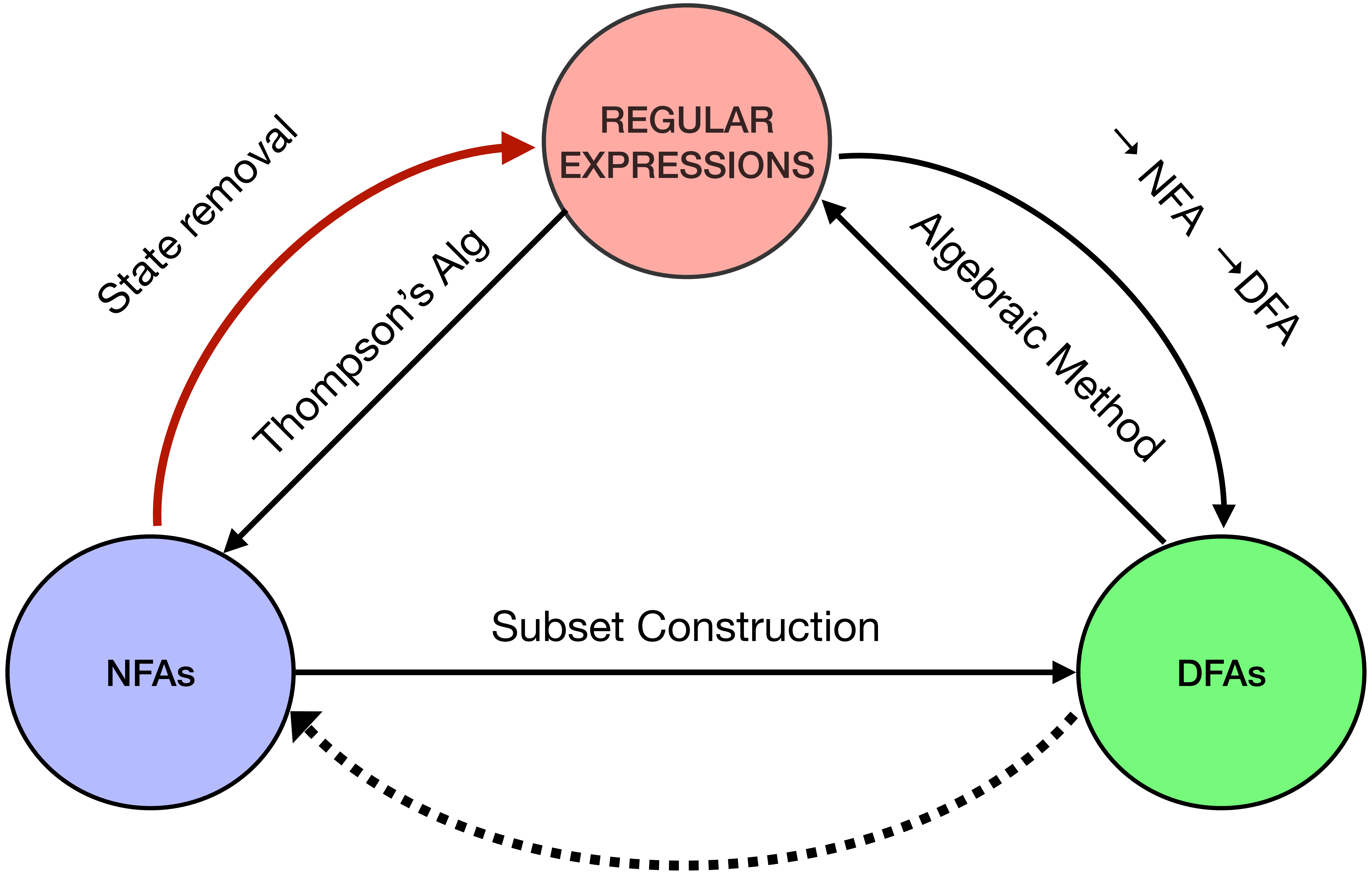
Equivalence of NFAs and Regular Expressions - State removal

Converting a DFA to Regular Expression

State removal

Key observation

If $q_1 = \delta(q_0, x)$ and $q_2 = \delta(q_1, y)$



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Converting a DFA to Regular Expression

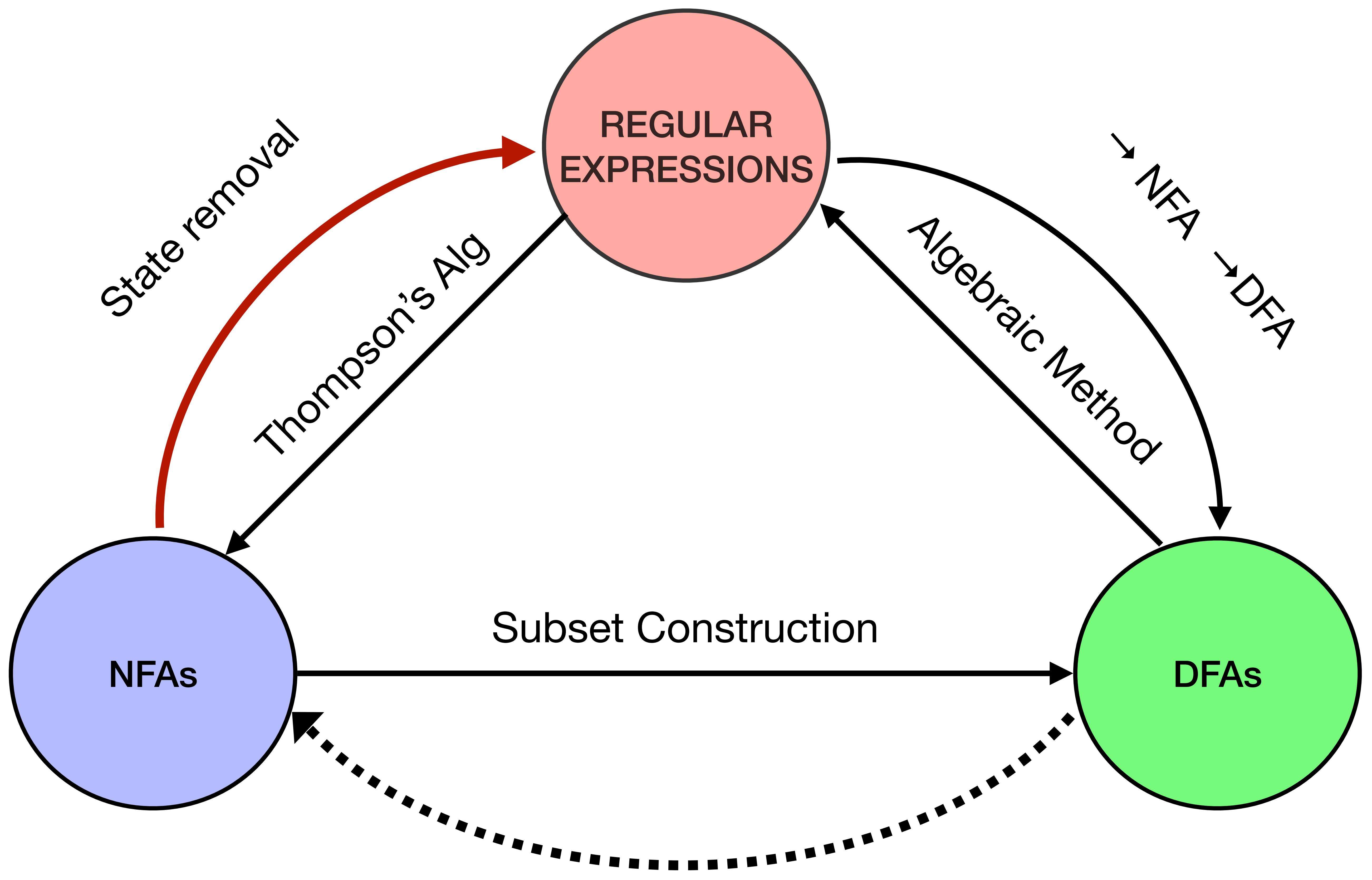
State removal

Key observation

If $q_1 = \delta(q_0, x)$ and $q_2 = \delta(q_1, y)$

then

$$q_2 = \delta(q_1, y) = \delta(\delta(q_0, x), y) = \delta(q_0, xy)$$



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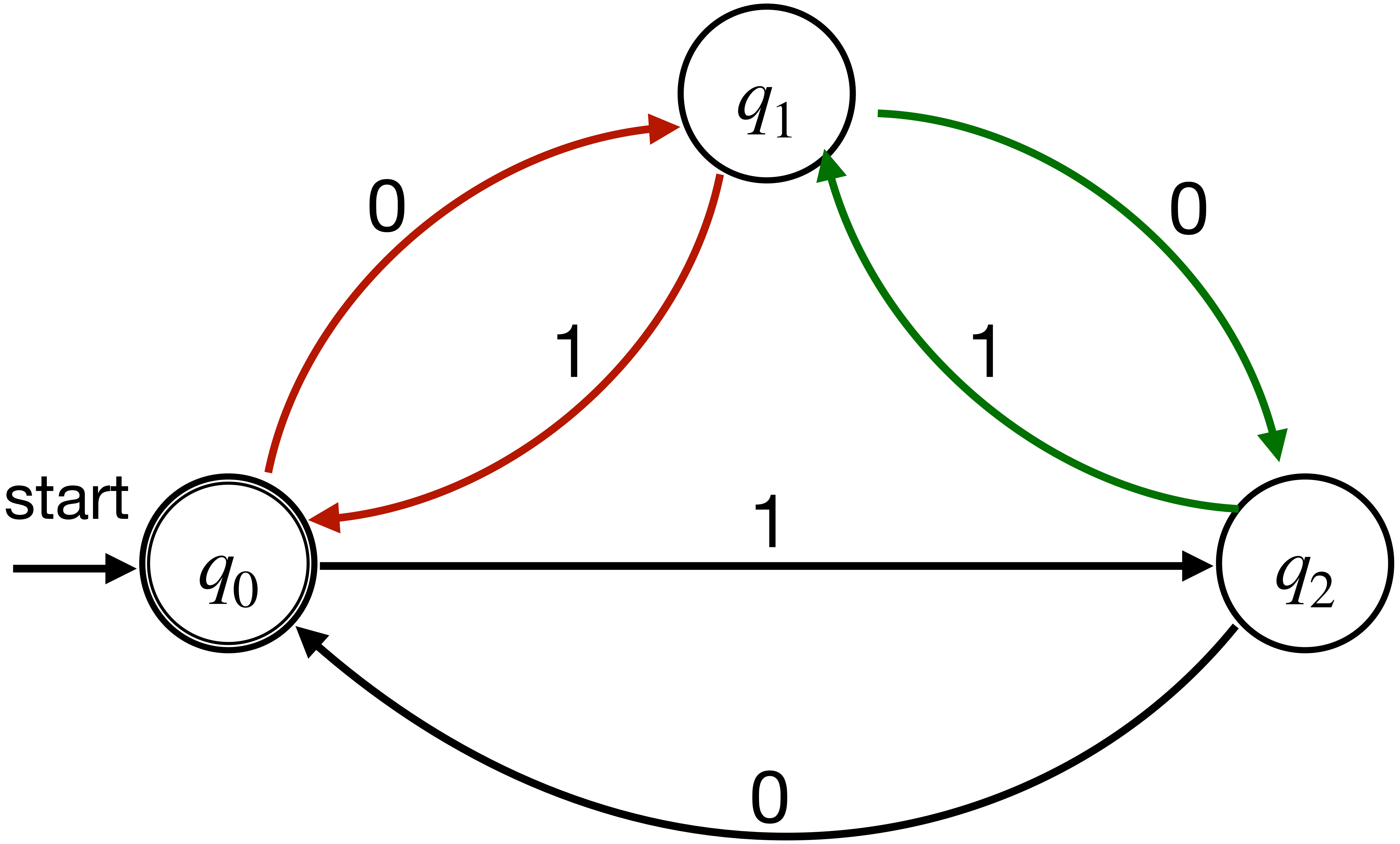
Converting a DFA to Regular Expression

State removal - example



$q_0 = \delta(q_1, 1)$

$q_1 = \delta(q_0, 0)$

$q_0 = \delta(\delta(q_0, 0), 1)$

$q_0 = \delta(q_0, 01)$

$q_2 = \delta(q_1, 0)$

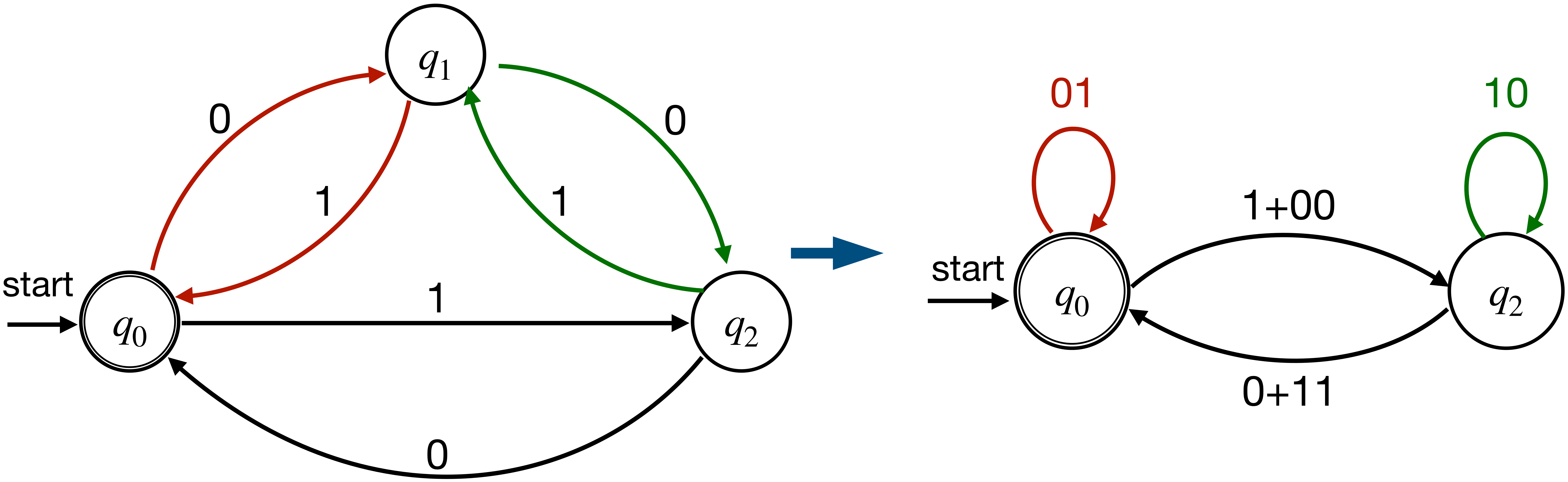
$q_1 = \delta(q_2, 1)$

$q_2 = \delta(\delta(q_2, 1), 0)$

$q_2 = \delta(q_1, 10)$

Converting a DFA to Regular Expression

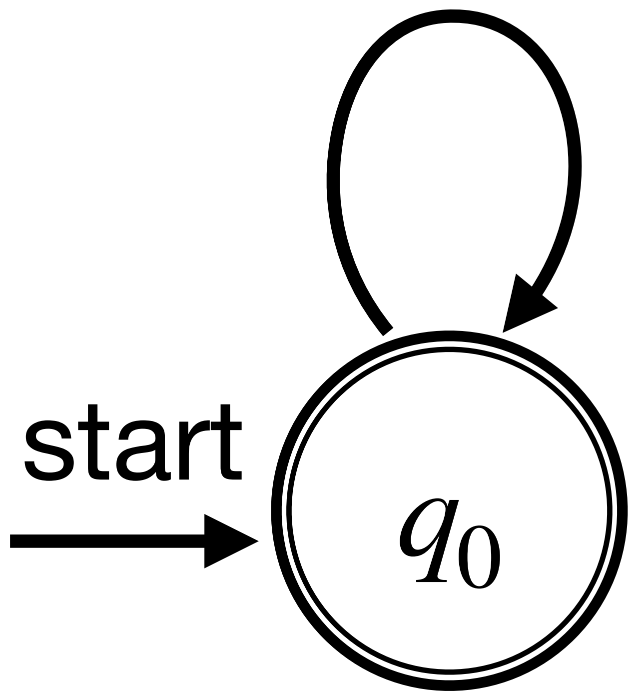
State removal - example

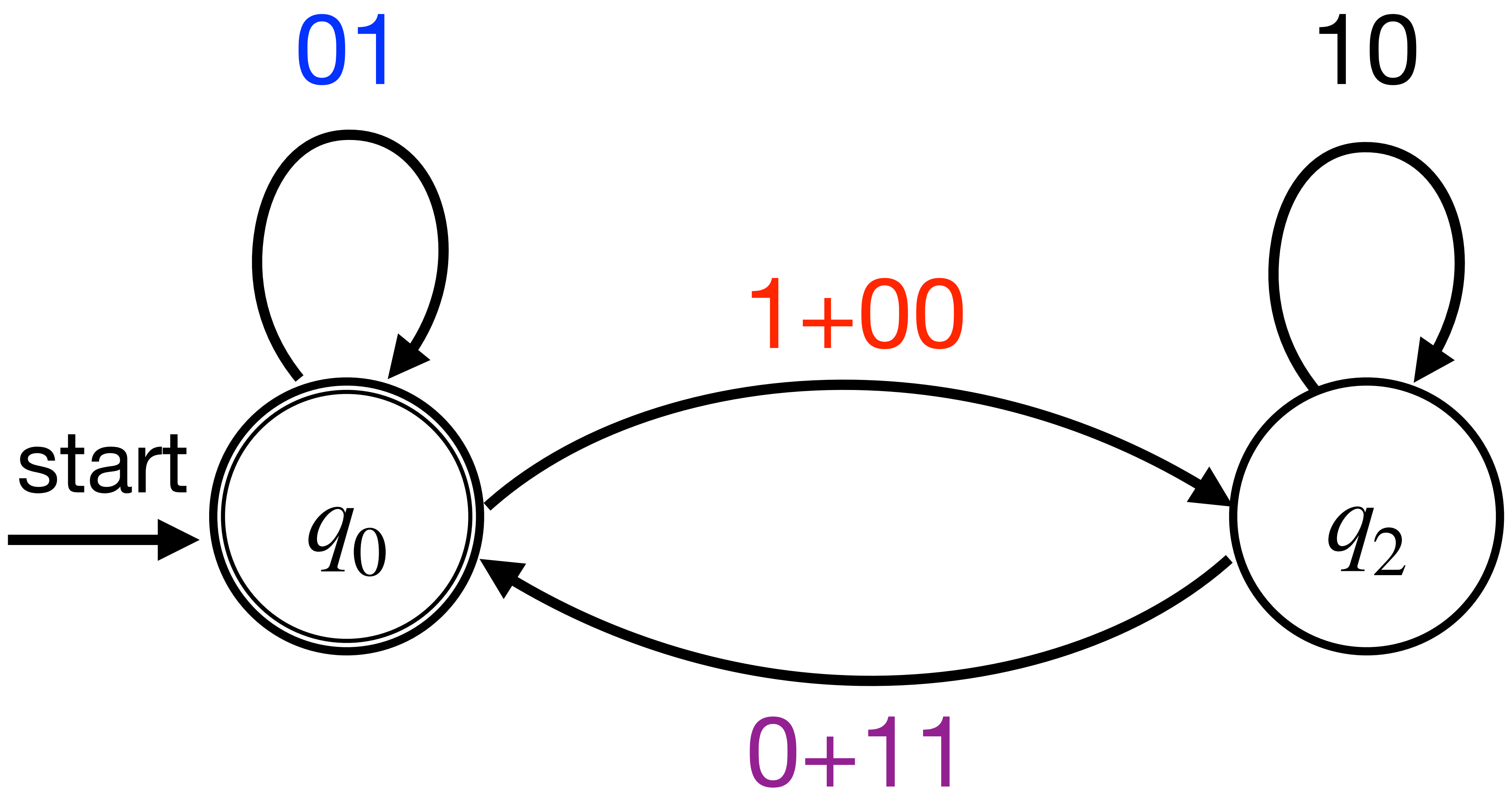


Converting a DFA to Regular Expression

State removal

$01 + (1 + 00)(10)^*(0 + 11)$





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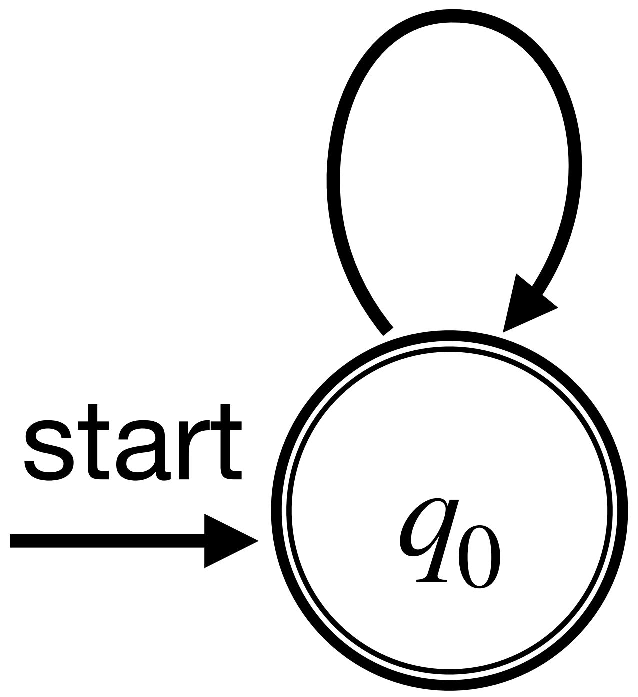


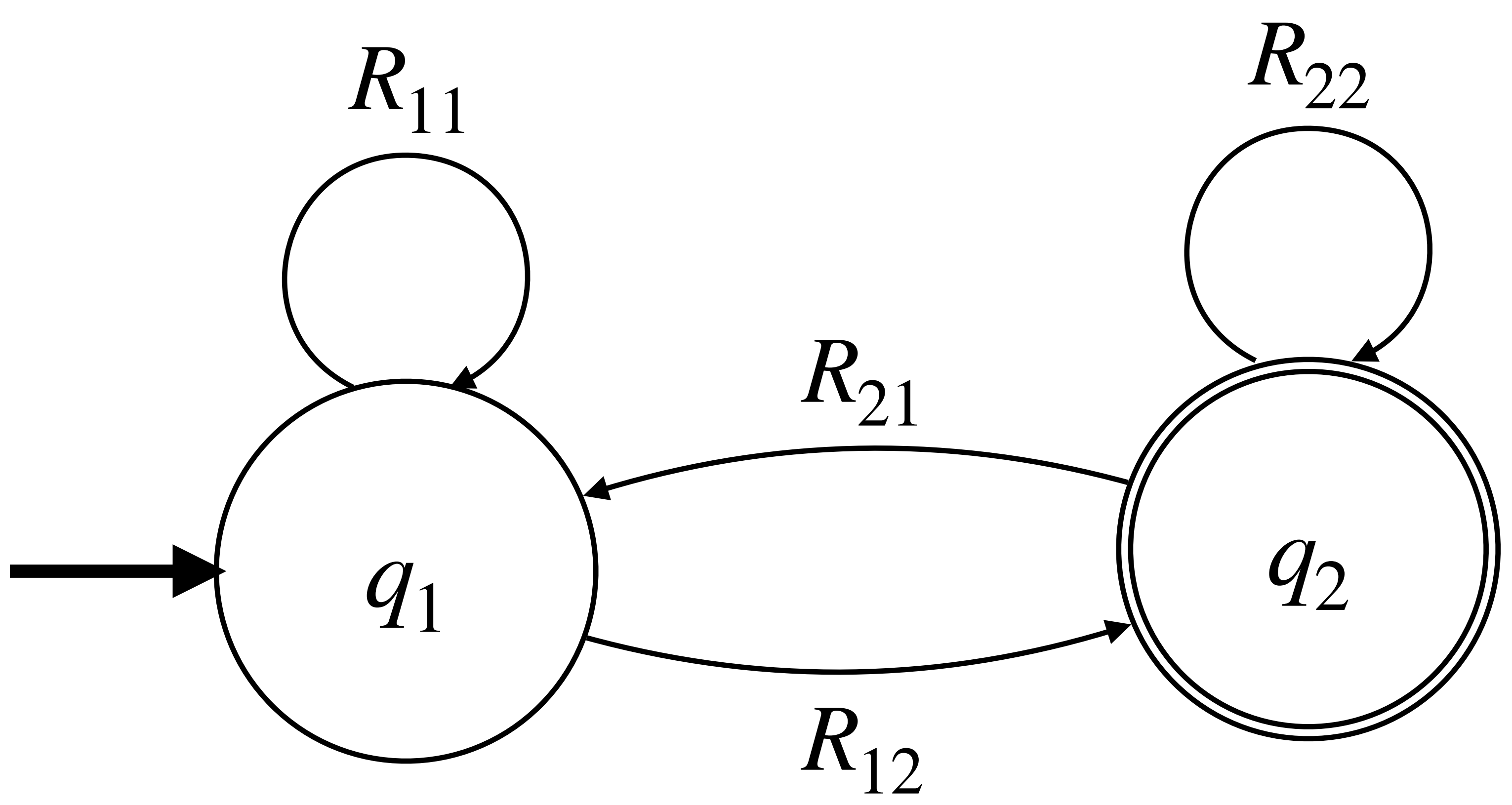
Final expression:

$$(01 + (1 + 00)(10)^*(0 + 11))^*$$

Converting a NFA to Regular Expression

State removal

- **Key idea:** We allow for a generalized NFA permitting arbitrary regular expression on the transition arrows.



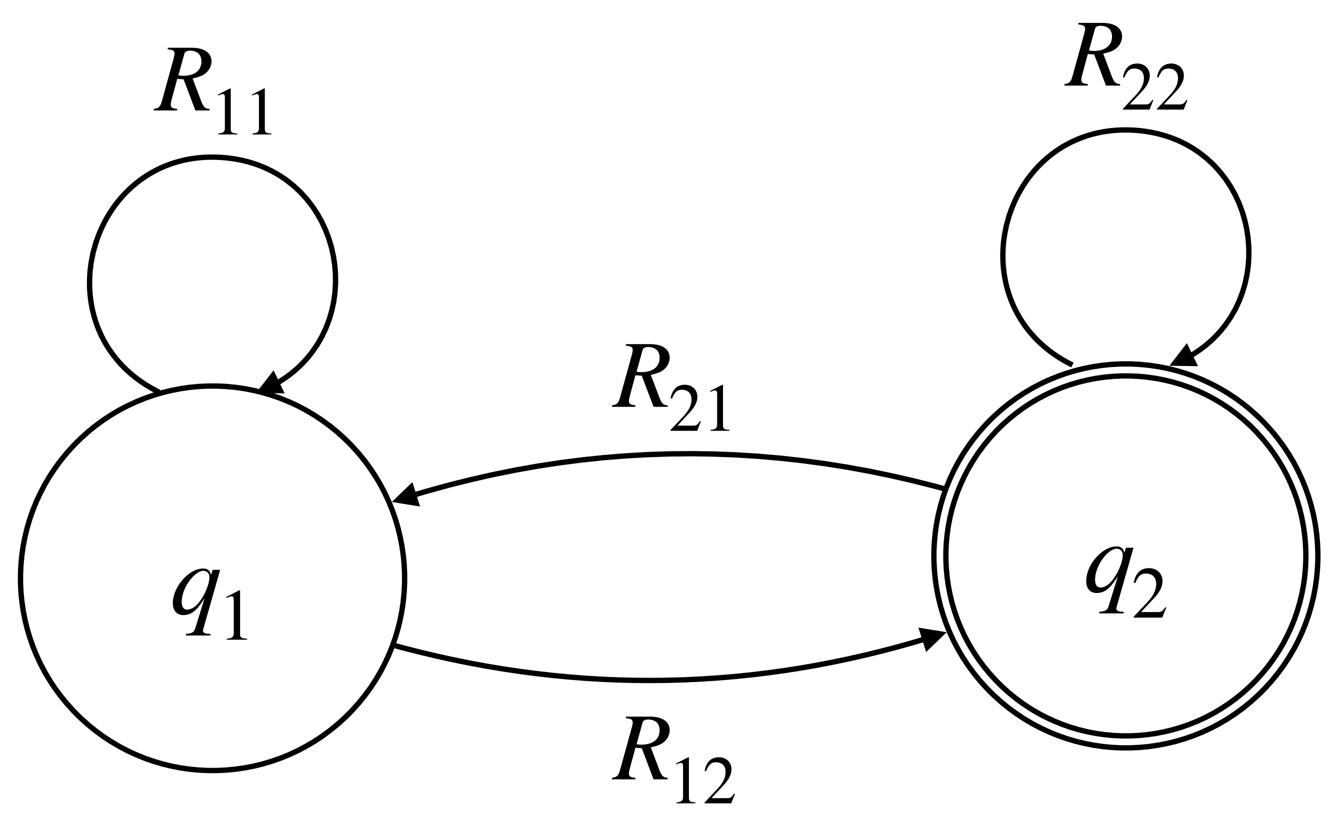
- R_{11}, R_{21}, R_{22}, R_{12} here are valid regular expressions.

- Can we eliminate states to get a clean RegEx?

Converting a NFA to Regular Expression

State removal

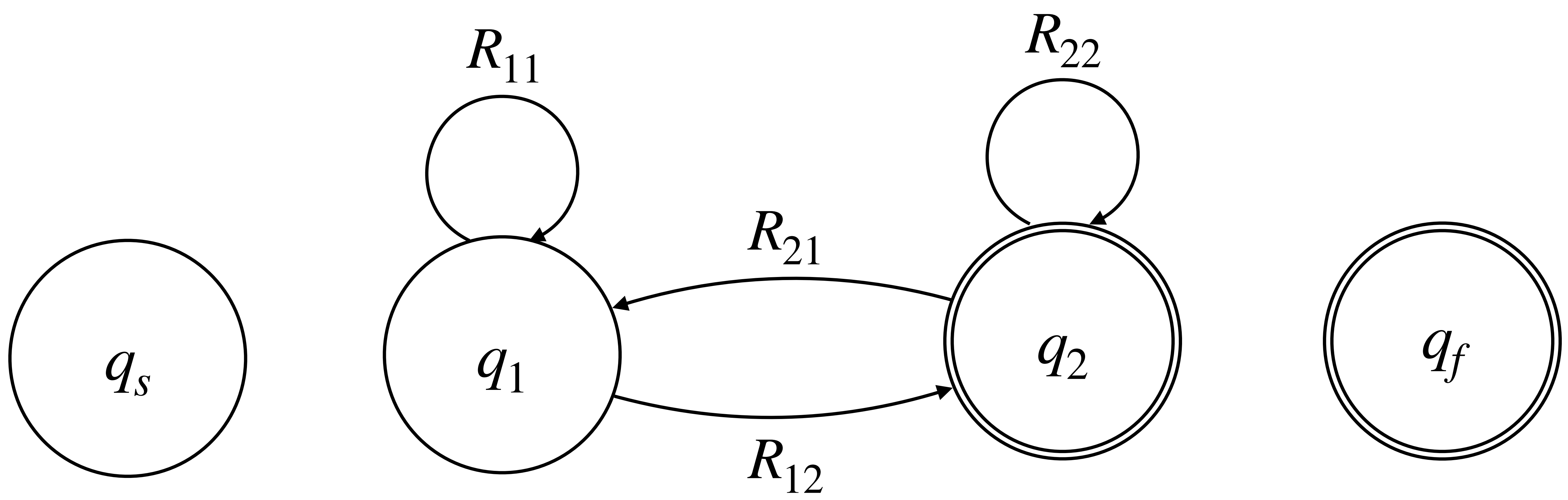
- **Step 1: Normalize**



Converting a NFA to Regular Expression

State removal

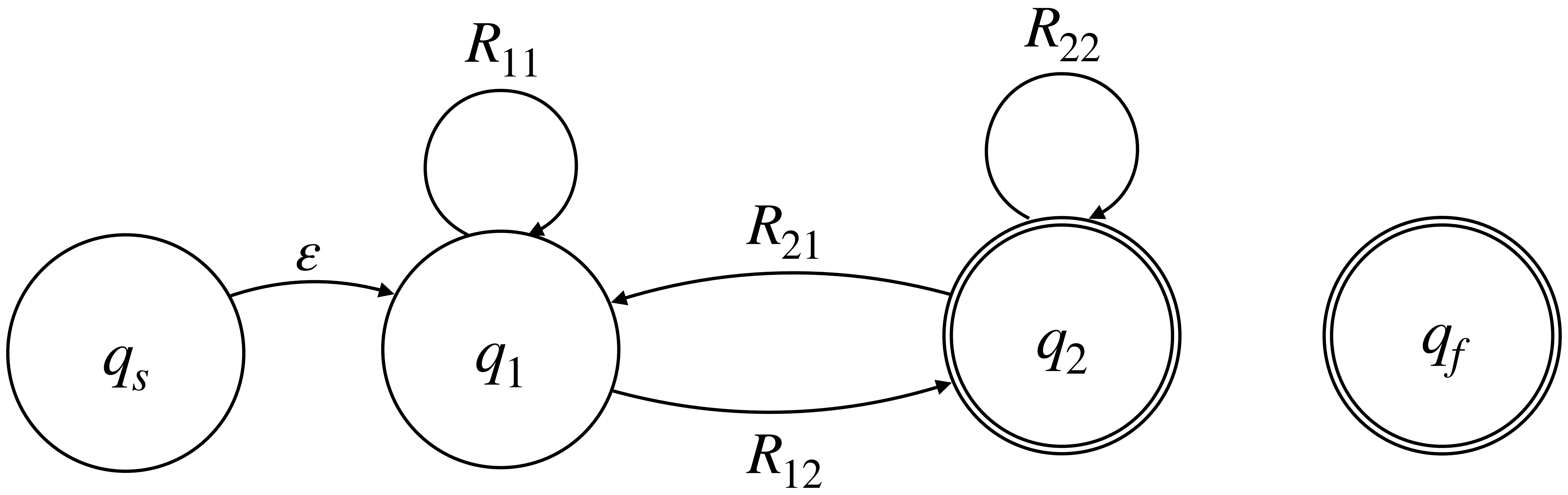
- **Step 1: Normalize**
  - Add a new start state q_s and accept state q_f to the NFA.



Converting a NFA to Regular Expression

State removal

- **Step 1: Normalize**
  - Add a new start state q_s and accept state q_f to the NFA.
  - Add an ε-transition from q_s to the old start state of N.



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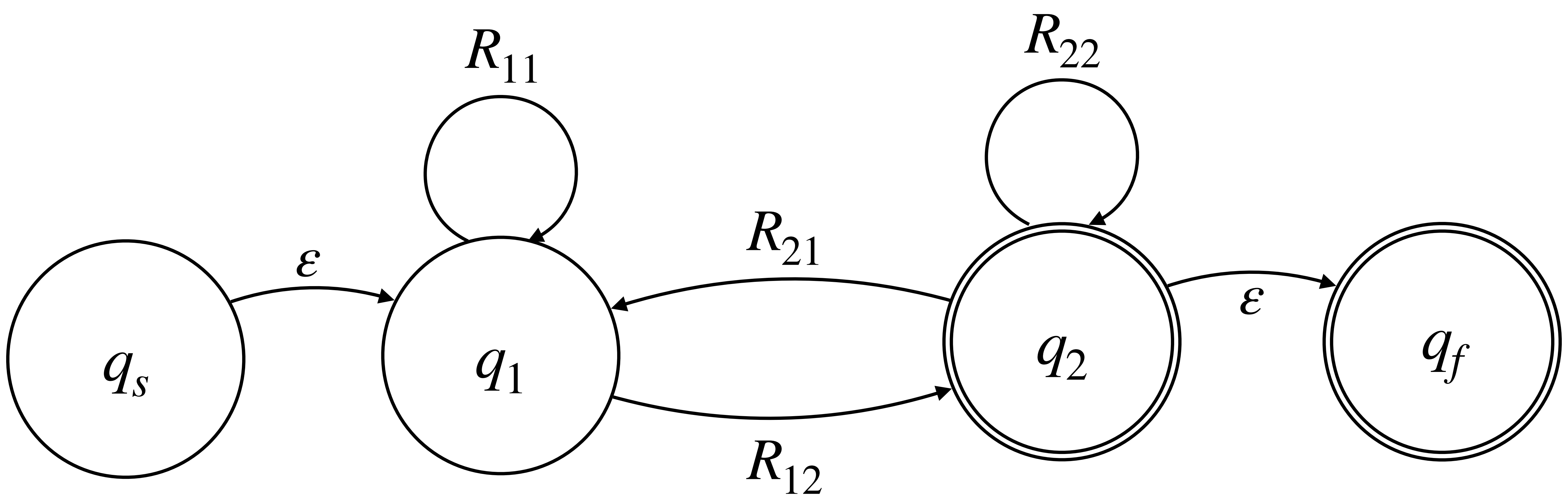

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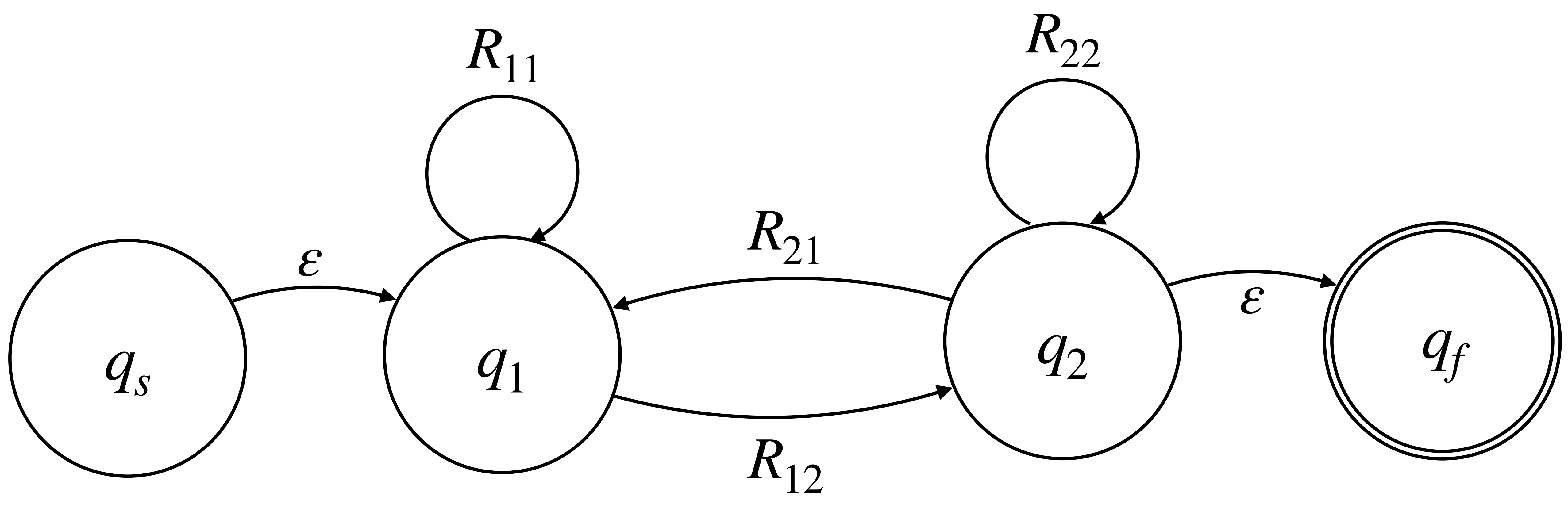
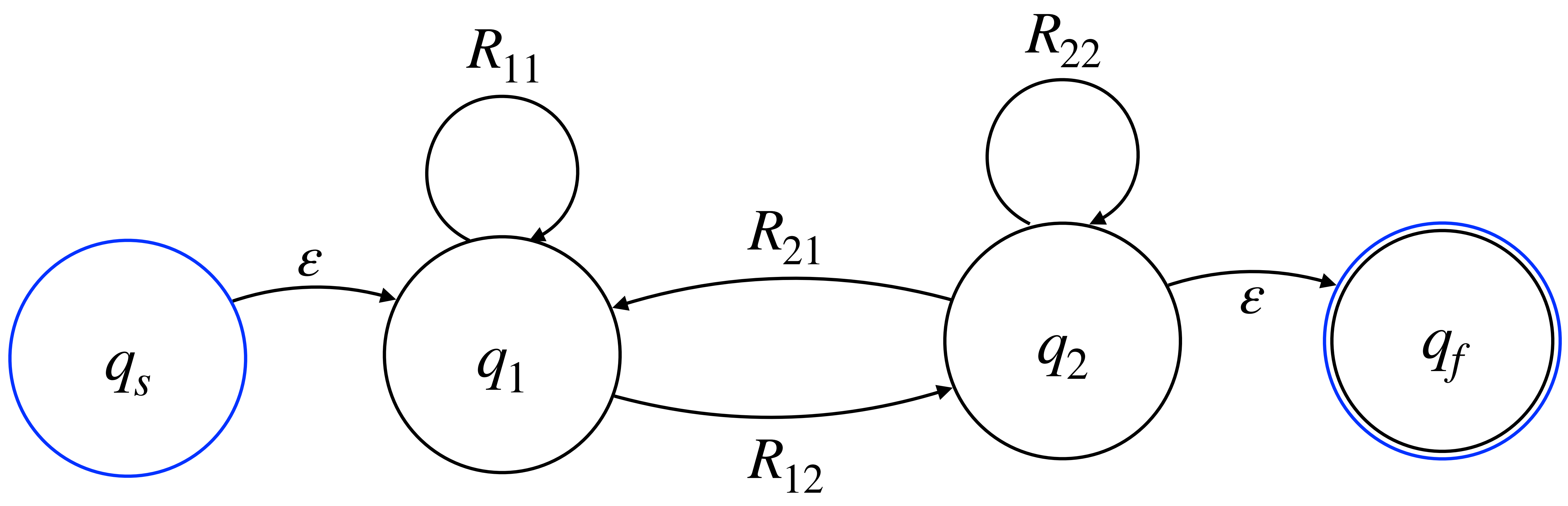


Converting a NFA to Regular Expression

State removal

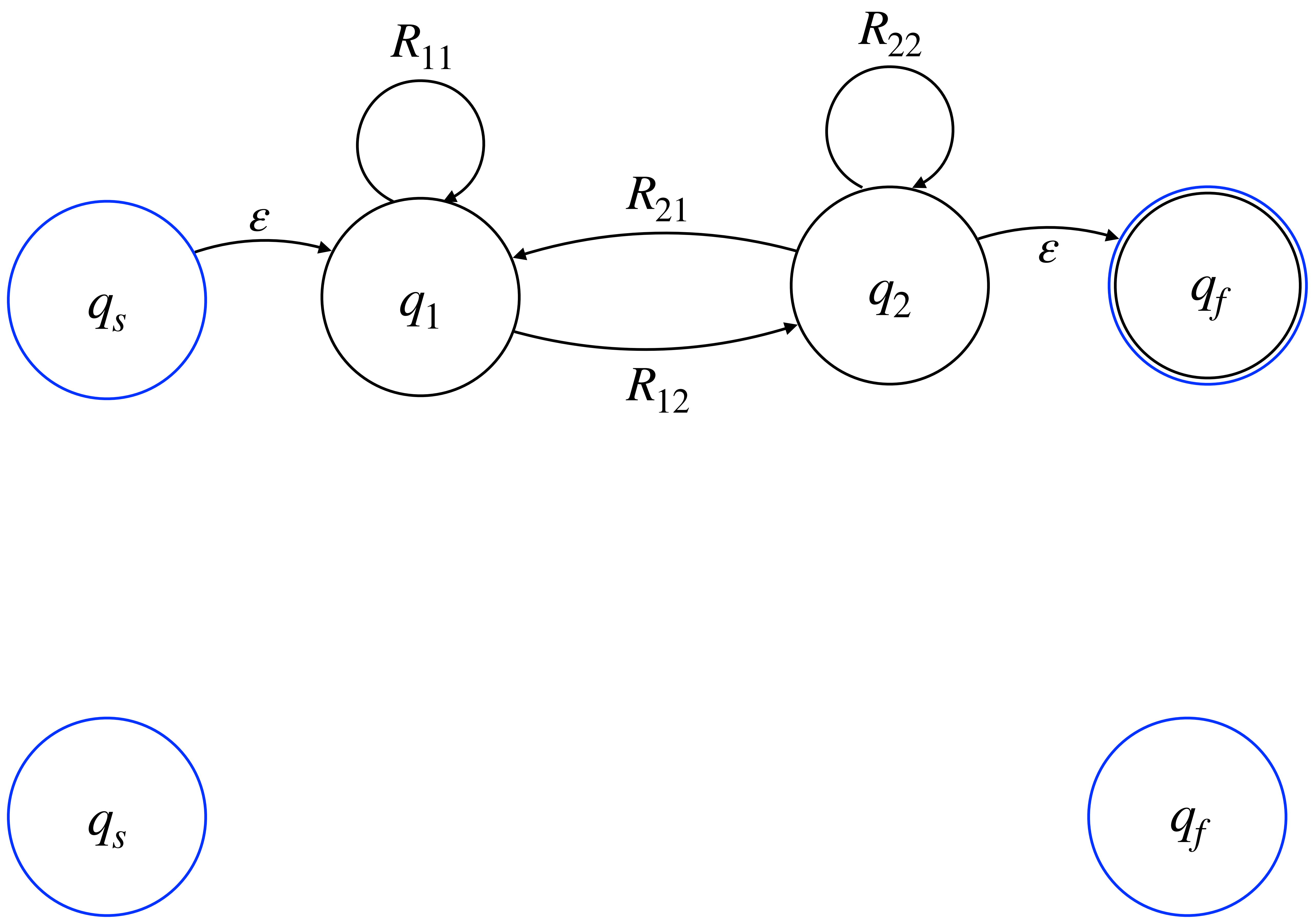
- **Step 2: Remove states**



Converting a NFA to Regular Expression

State removal

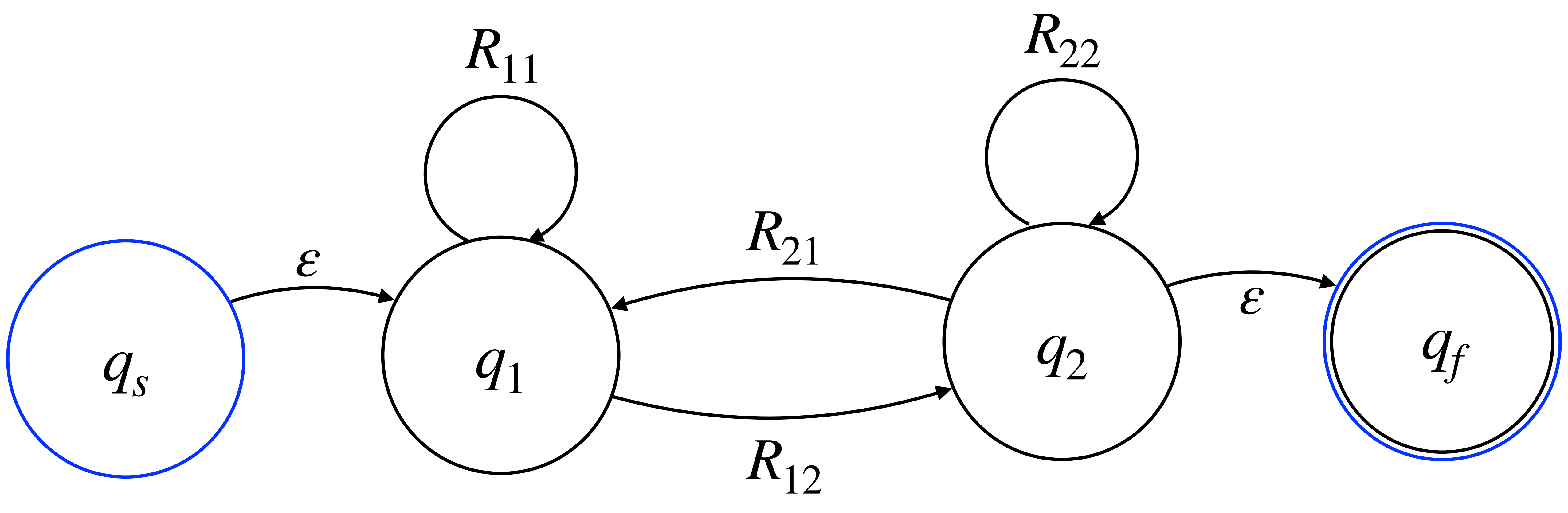
- **Step 2: Remove states**
  - Repeatedly remove states other than q_s and q_f from the NFA by “shortcutting” them until only two states remain: q_s and q_f.

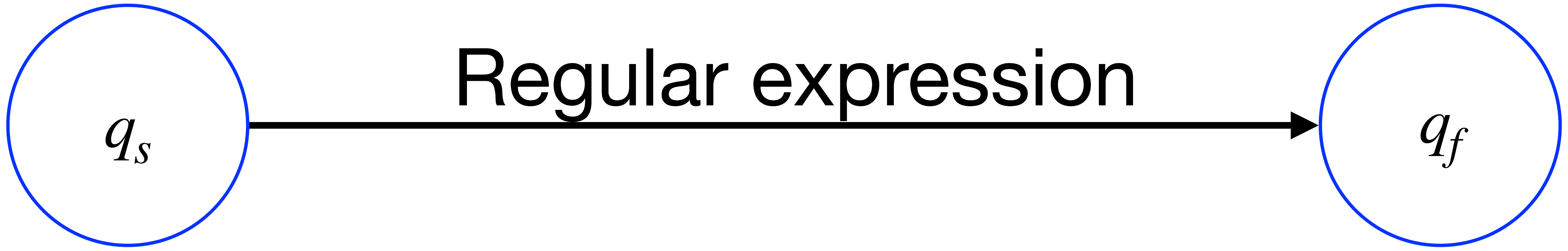


Converting a NFA to Regular Expression

State removal

- **Step 2: Remove states**
  - Repeatedly remove states other than q_s and q_f from the NFA by "shortcutting" them until only two states remain: q_s and q_f.
  - The transition from q_s to q_f is then a regular expression for the NFA.





Converting a NFA to Regular Expression

State removal

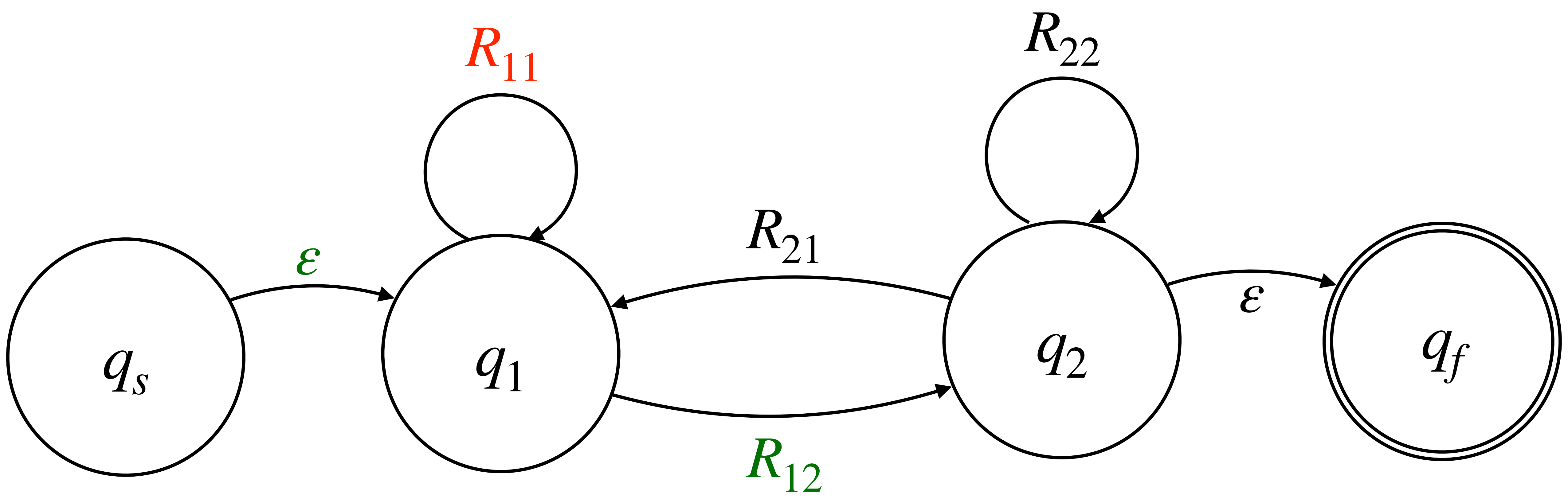
- **Step 2: Details**
  - For each pair (q_1, q_2) such that

$$q_1 \xrightarrow{R_{in}} q, \;\; q \xrightarrow{R_{out}} q_2$$

Add a transition such that

$$q_2 = \delta\left(q_1, R_{in} \cdot R_q^* \cdot R_{out}\right)$$

where R_q is a self-transition (if any)



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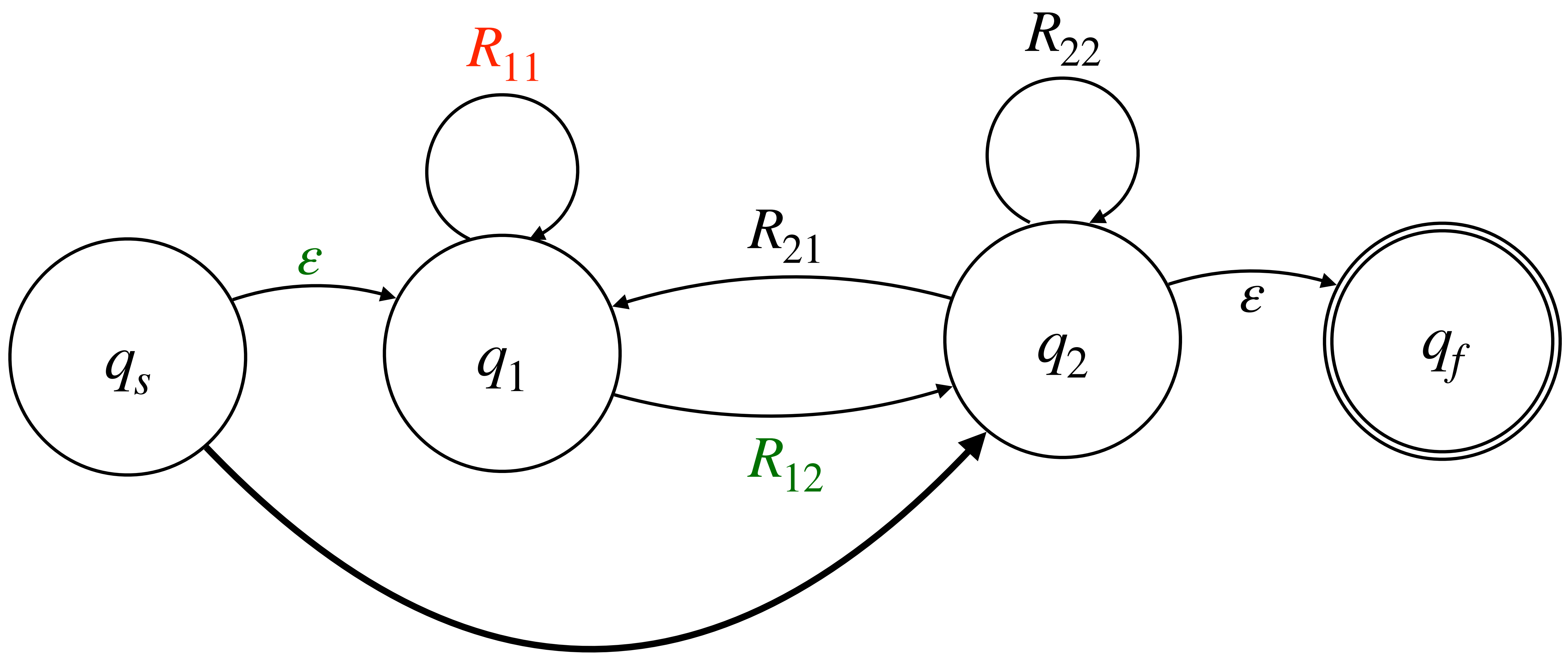

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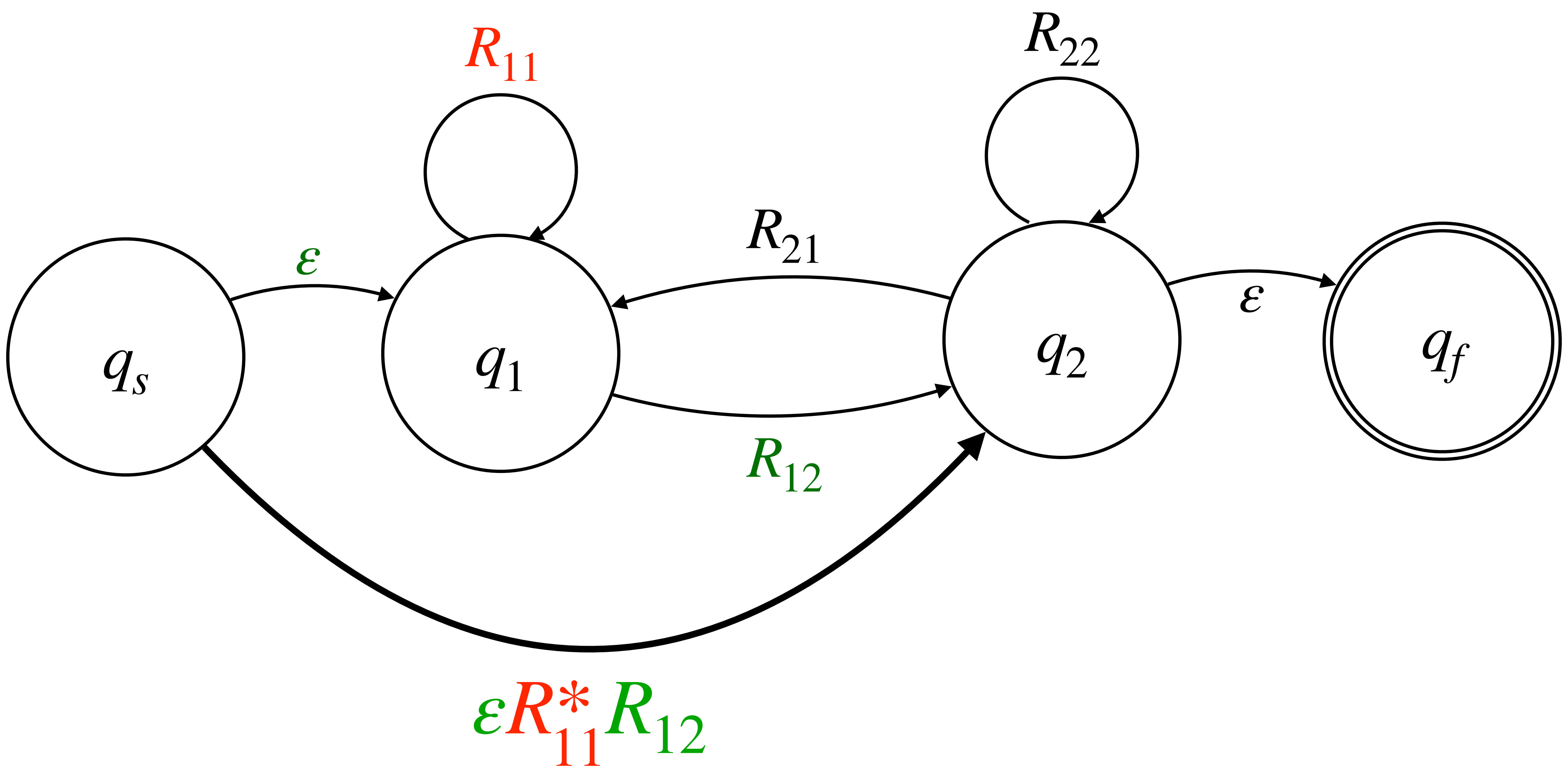

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State removal

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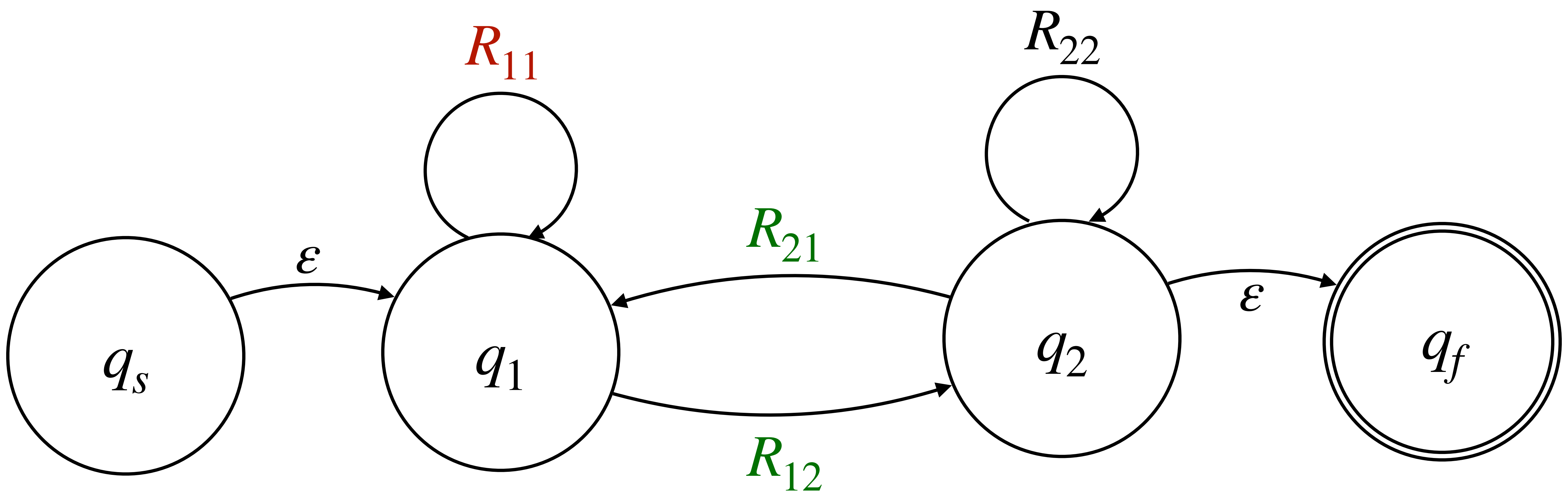
  - For each pair (q_1, q_2) such that

$$q_1 \xrightarrow{R_{in}} q, \;\; q \xrightarrow{R_{out}} q_2$$

Add a transition such that

$$q_2 = \delta\left(q_1, R_{in} \cdot R_q^* \cdot R_{out}\right)$$

where R_q is a self-transition (if any)



Converting a NFA to Regular Expression

State removal

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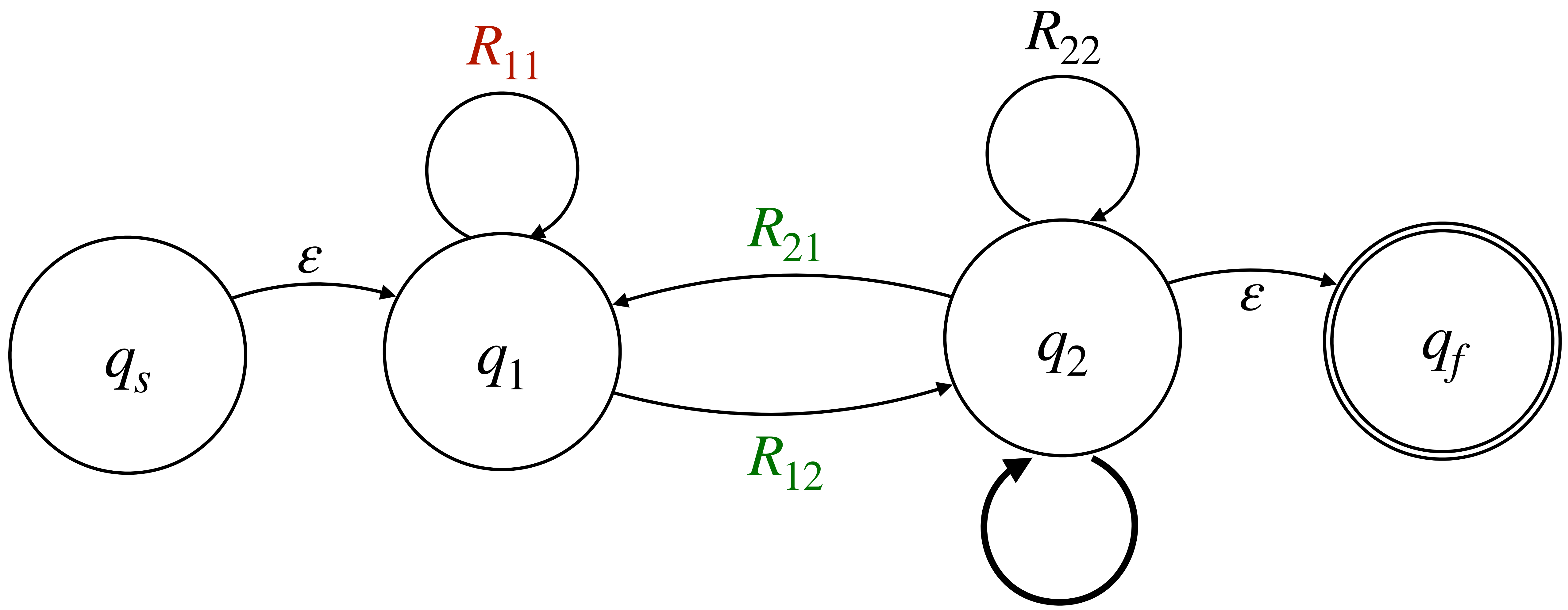

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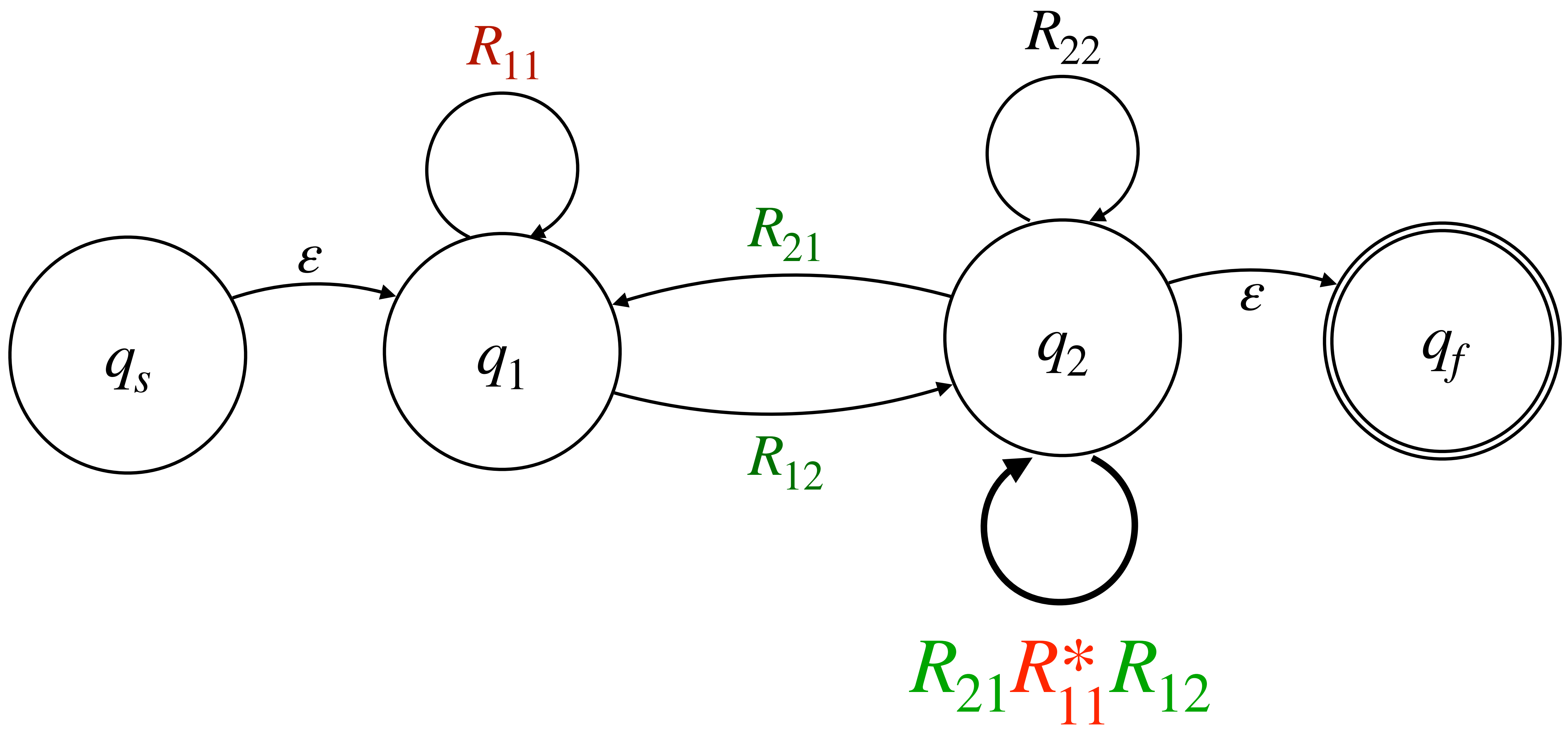

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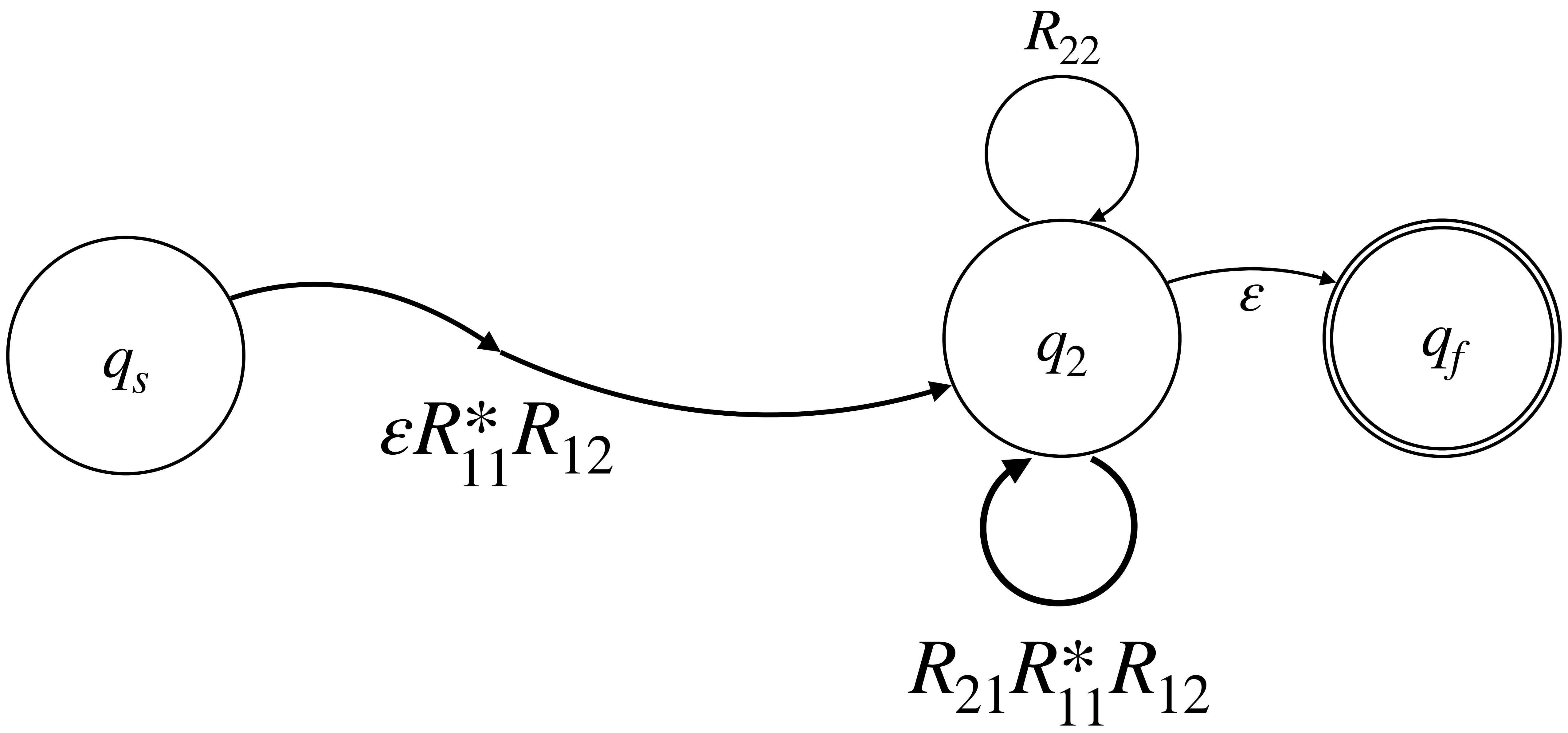

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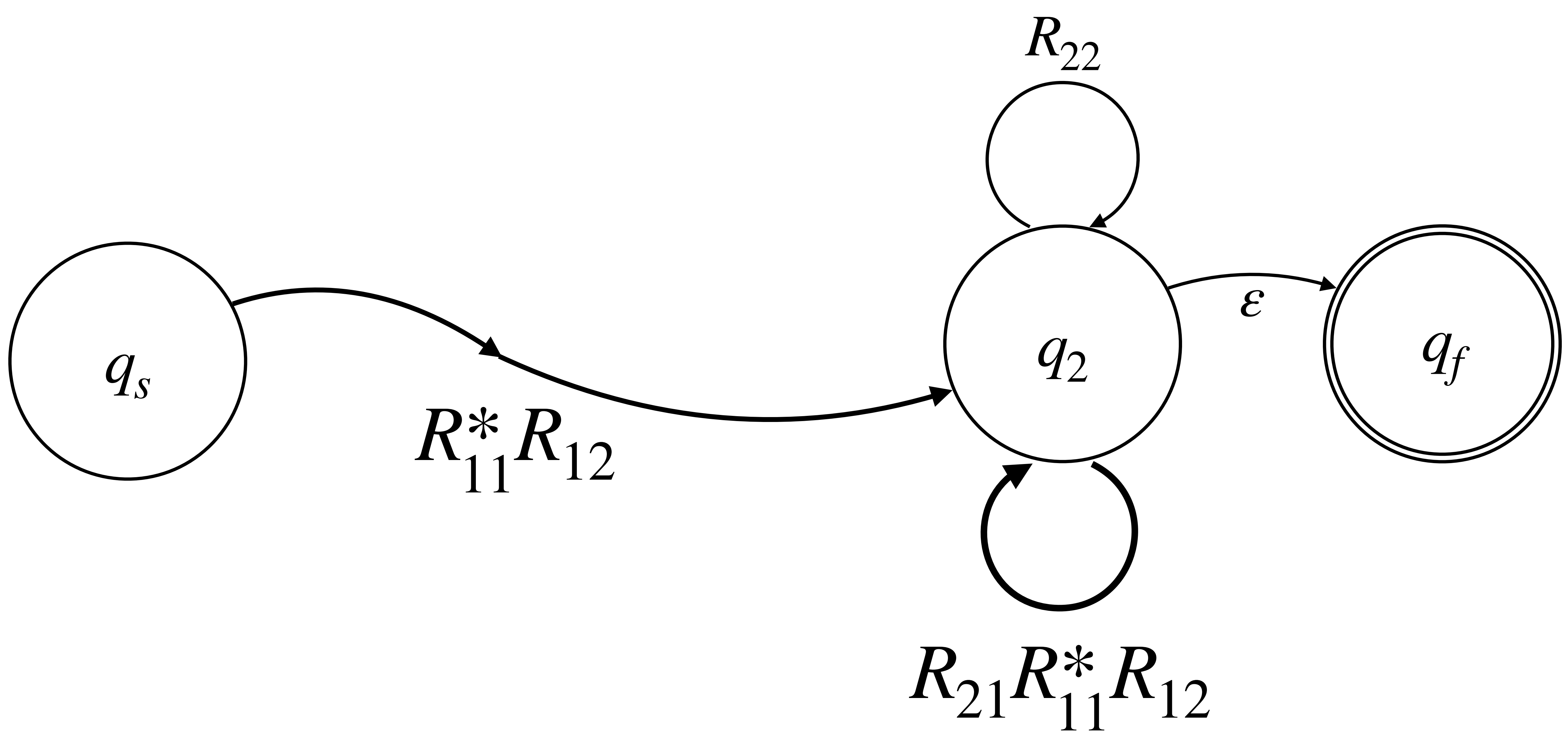
  - Use union operation to handle multiple transitions



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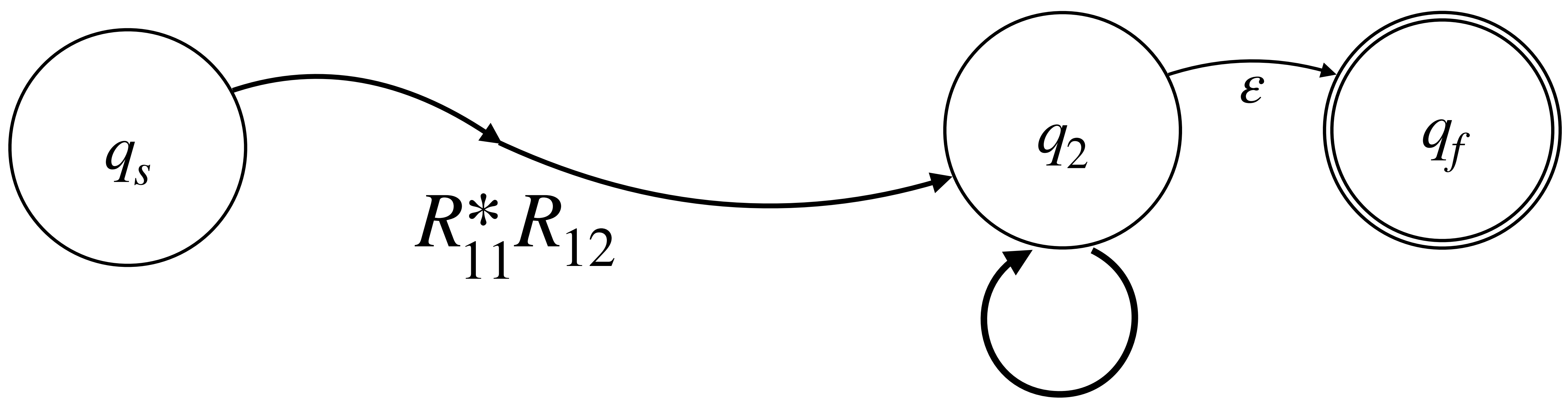

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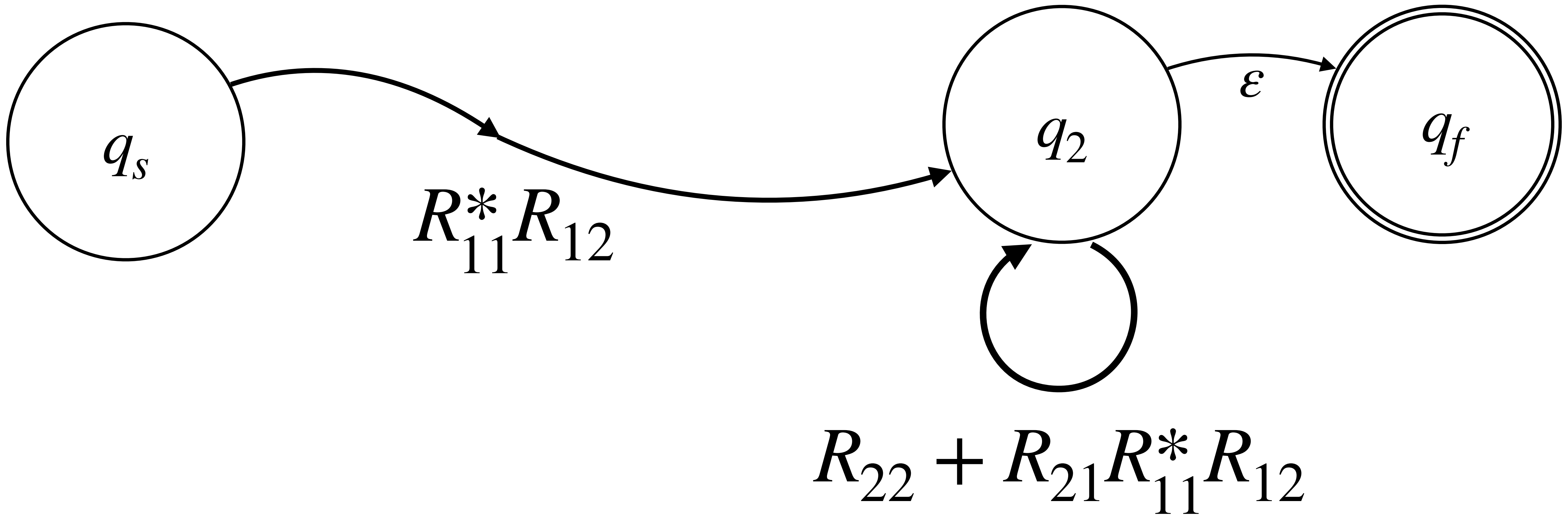

Converting a NFA to Regular Expression

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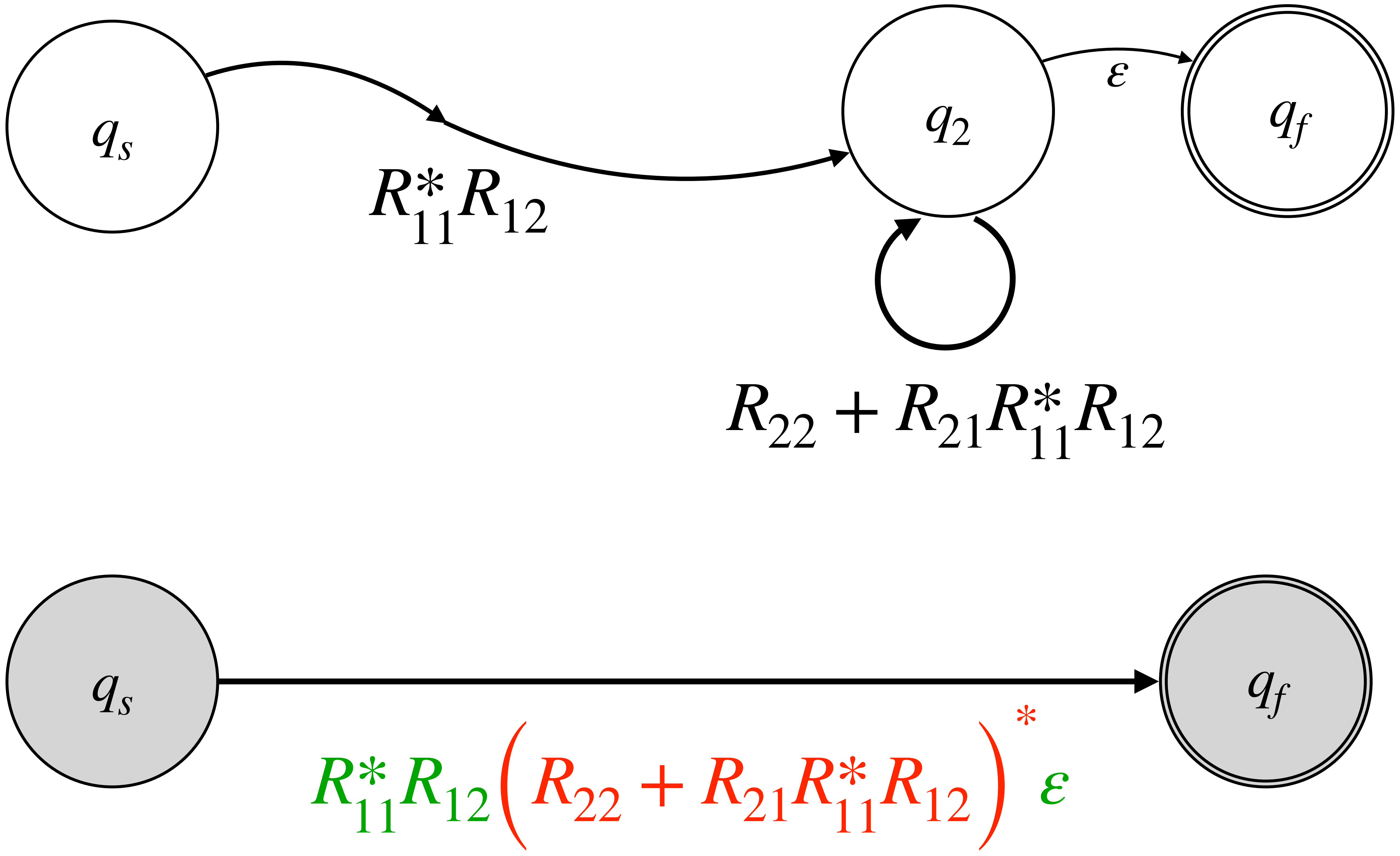

Equivalence of NFAs and Regular Expressions - Thompson's algorithm

NFA from a RegEx

Thompson's algorithm

- **Key idea:** Represent regular operations (Union, Concatenation & Kleene Star) using NFAs.

NFA from a RegEx

Thompson's algorithm

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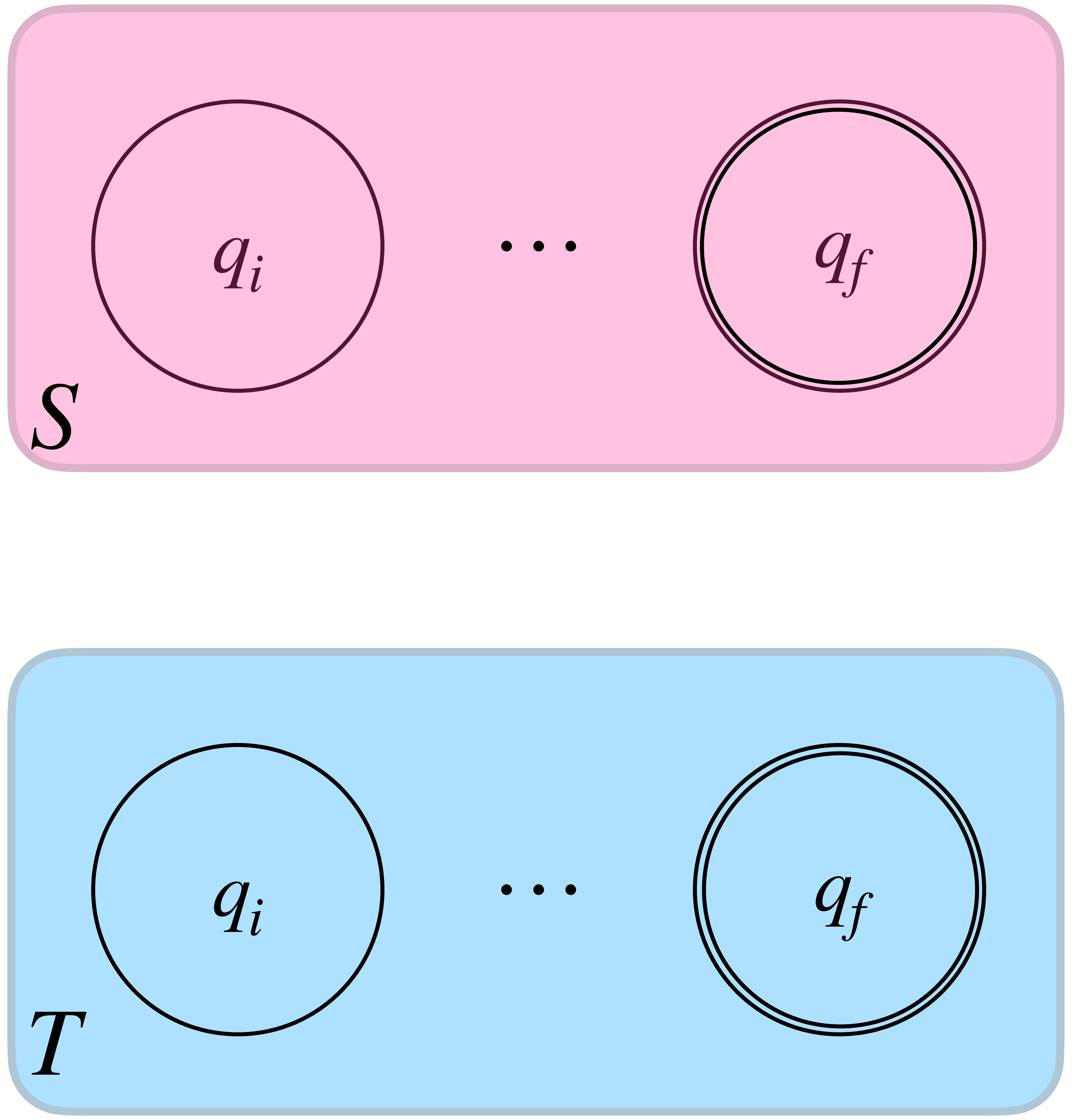
- Given: Two NFAs S and T representing languages L_S and L_T

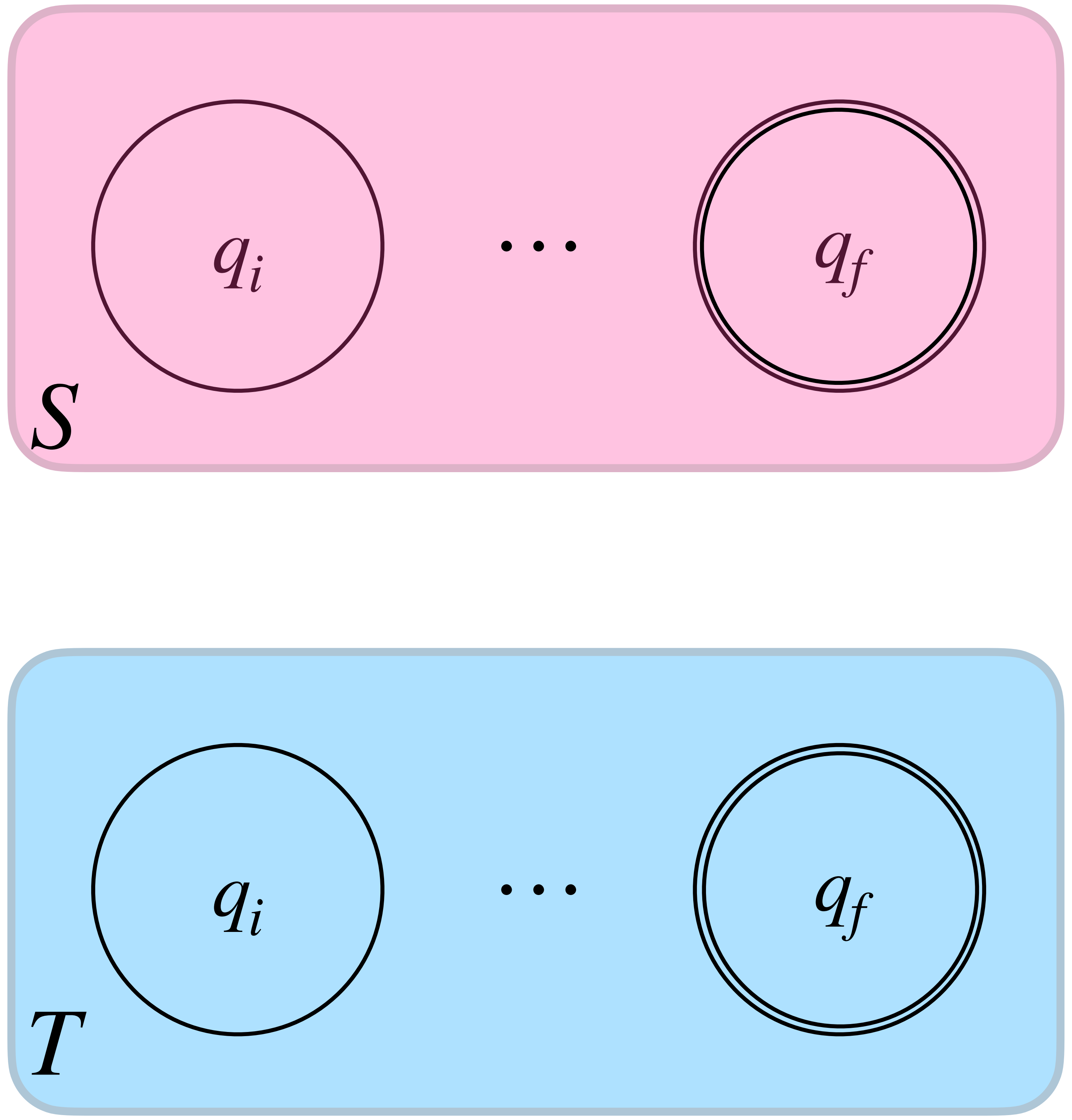


NFA from a RegEx

Thompson's algorithm

- **Key idea:** Represent regular operations (Union, Concatenation & Kleene Star) using NFAs.

- Given: Two NFAs S and T representing languages L_S and L_T
    - What NFA represents $L_S \cdot L_T$, $L_S + L_T$ and L_S^*

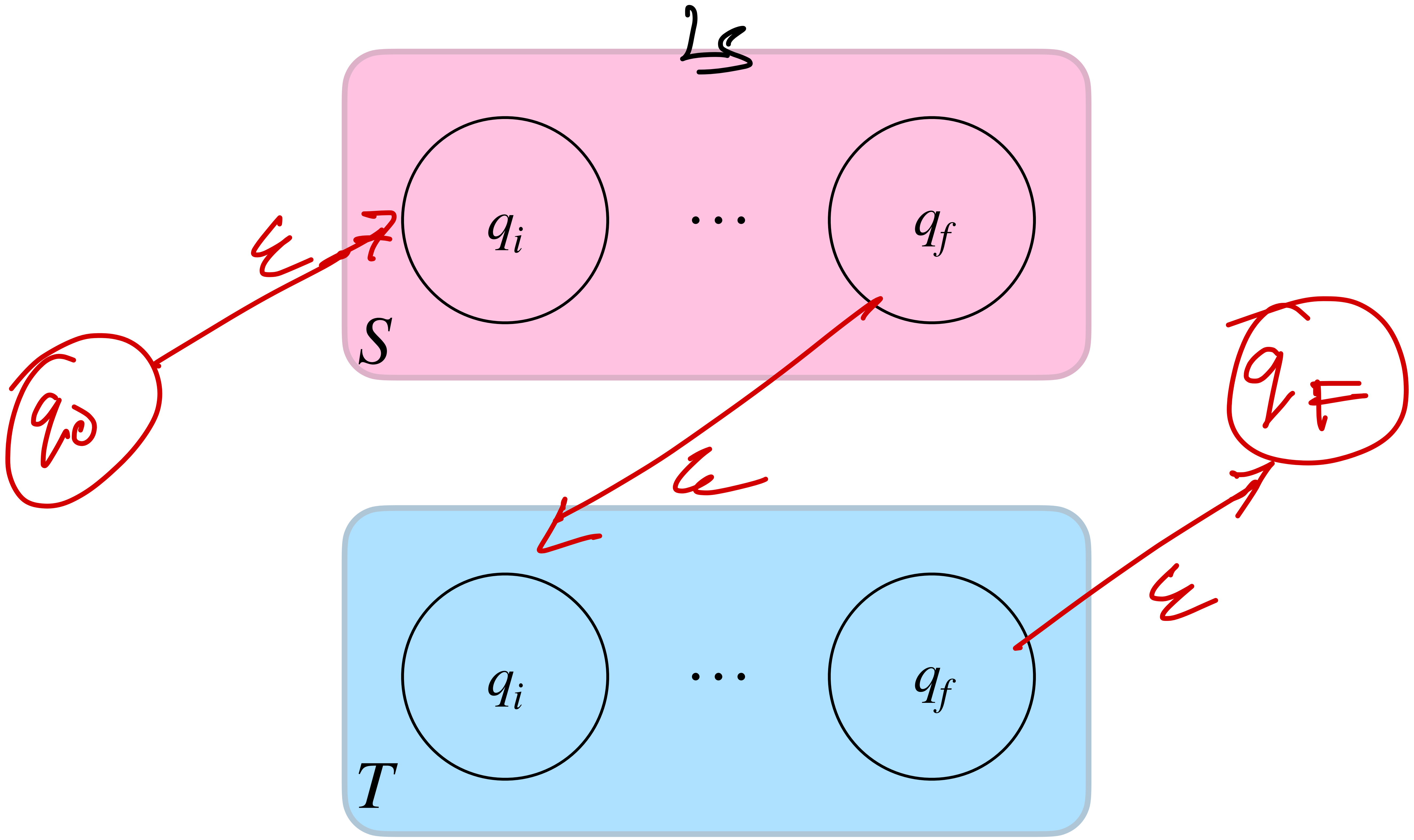


NFA from a RegEx

Regular operation rules

- **Concatenation**
- $\mathbf{L} = L_s \cdot L_t$

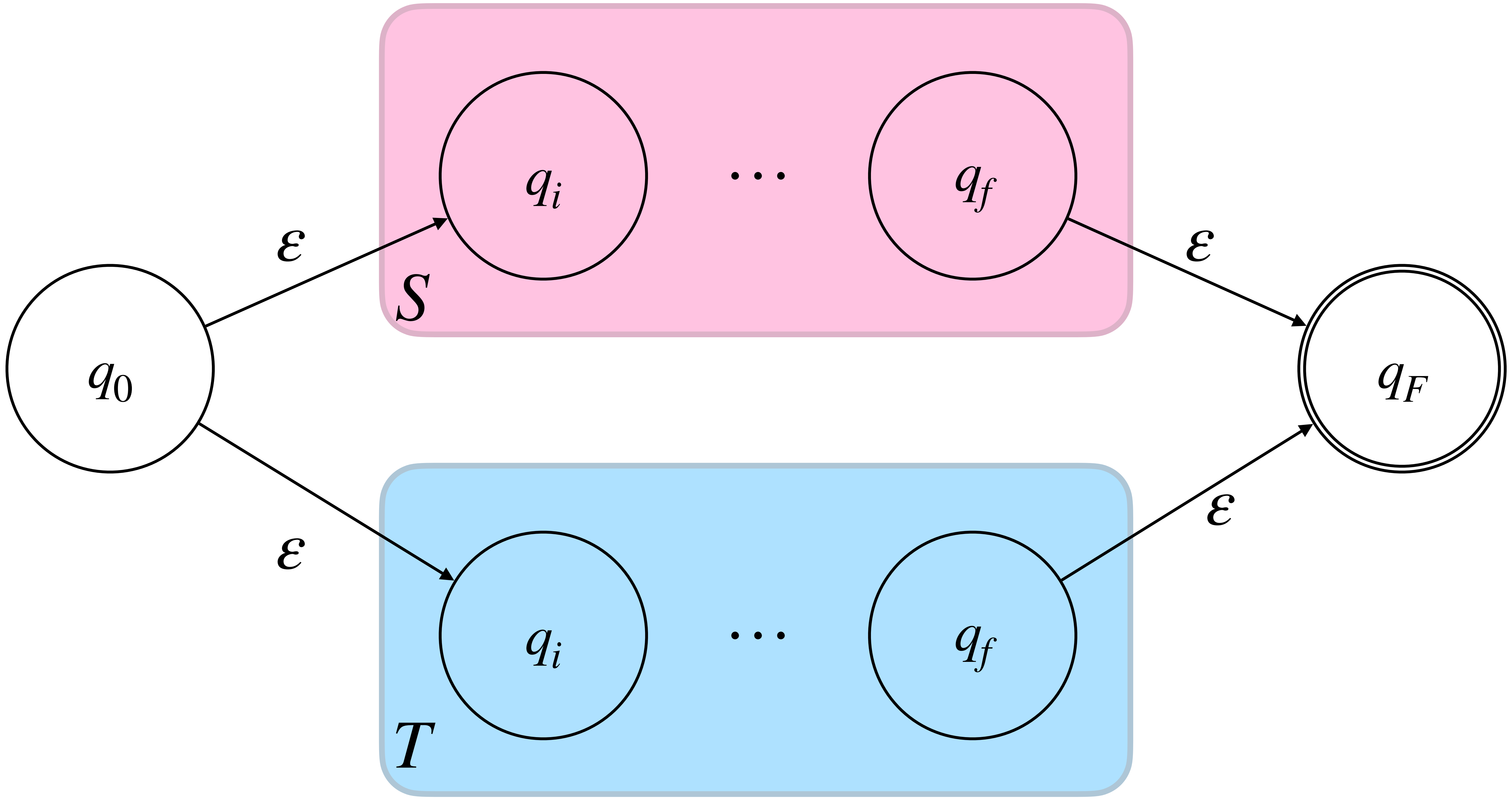
"series connection"



NFA from a RegEx

Regular operation rules

- **Union**
- $\mathbf{L} = L_S + L_T$
  - "Parallel connection"



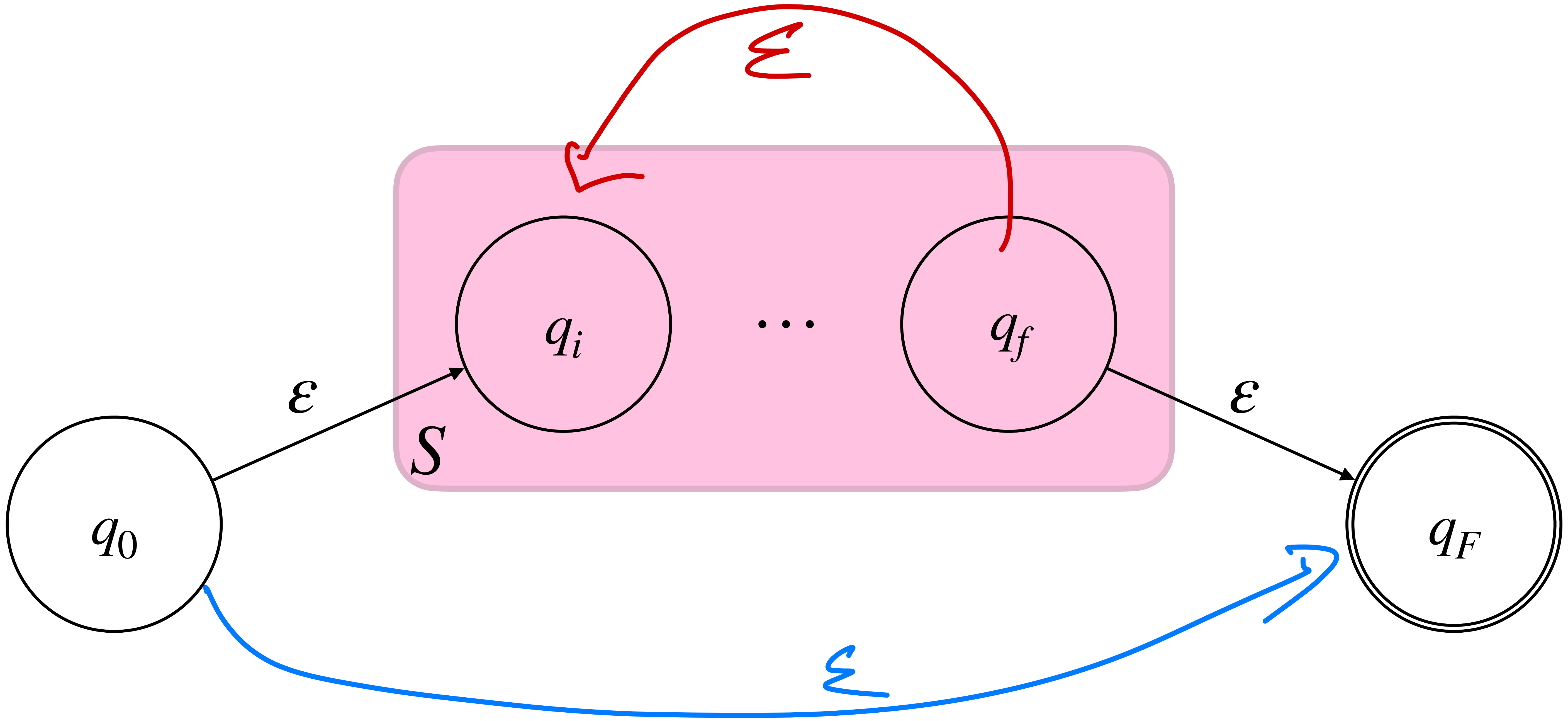
NFA from a RegEx

Regular operation rules

- **Kleene star**
- $\mathbf{L} = L_s^*$

should include

- empty string
- repetitions.



NFA from a RegEx

Example

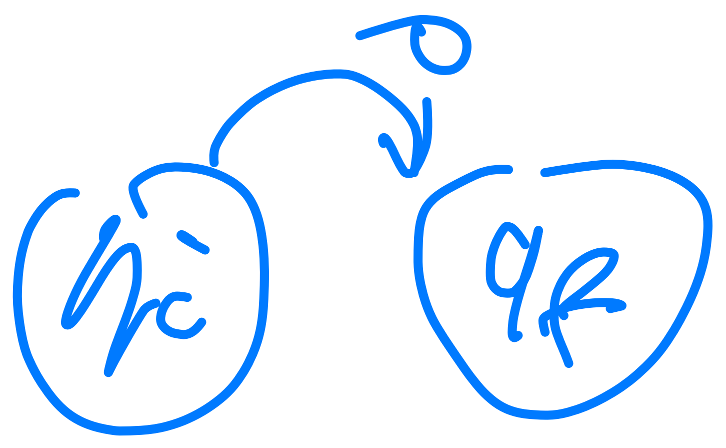
- **Find an NFA for** $(0+1)^*(101+010)(0+1)^*$

  $N_A \quad N_C \quad N_D \quad N_A$

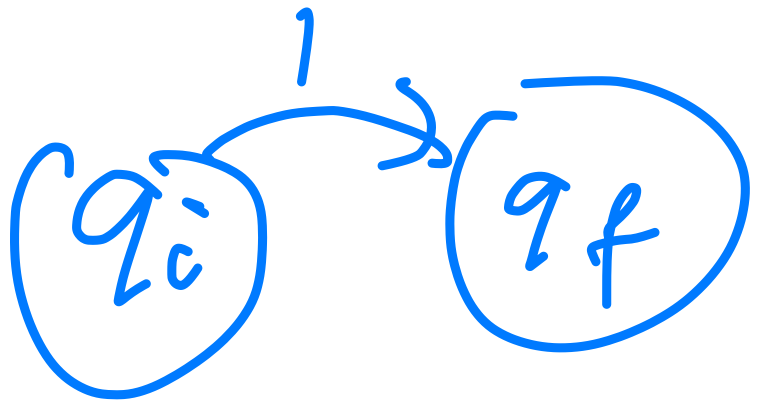
- Rewrite:

$N_A = N_0 + N_1$

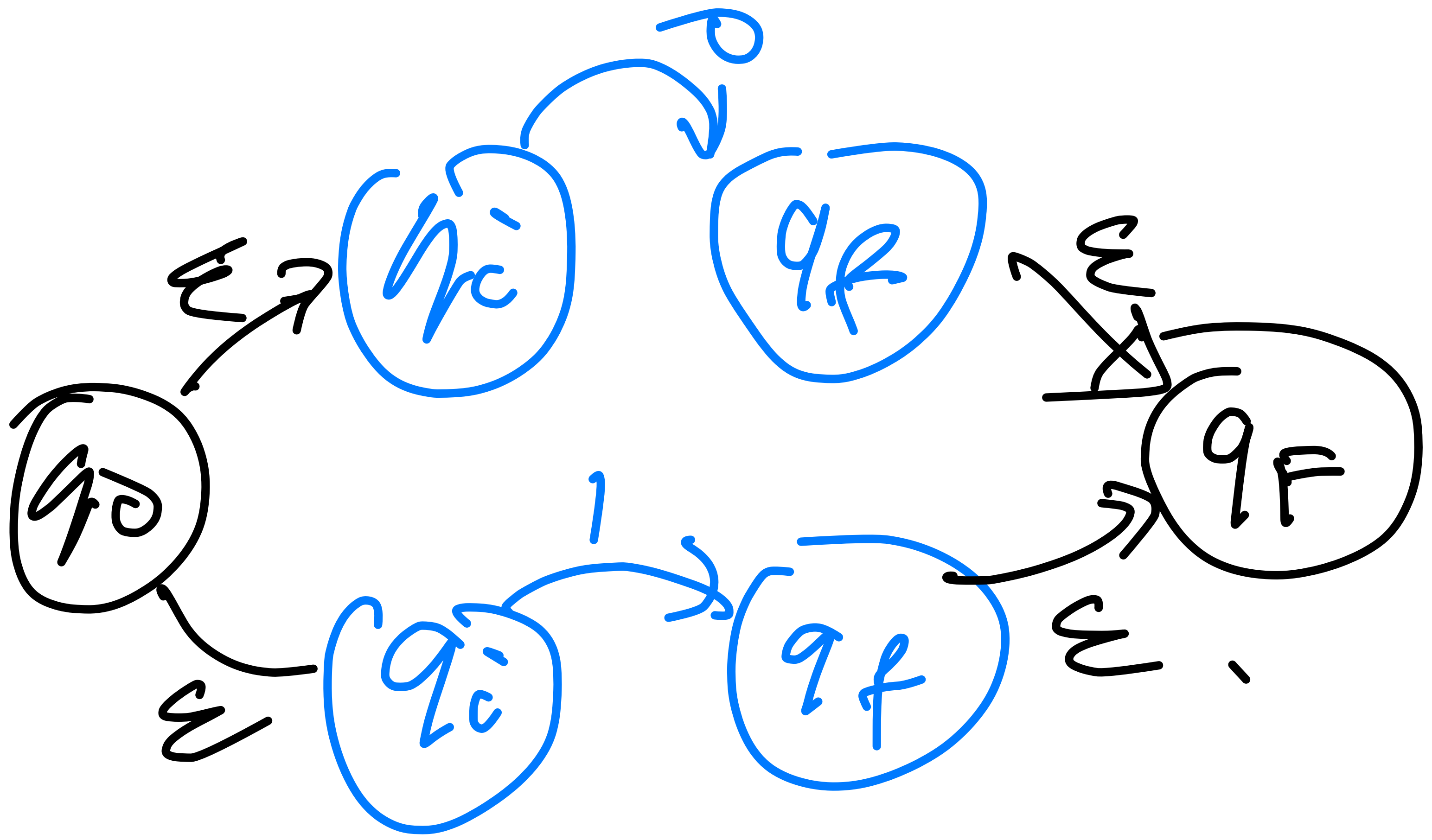
N_0 :



N_1 :



$N_0 + N_1$



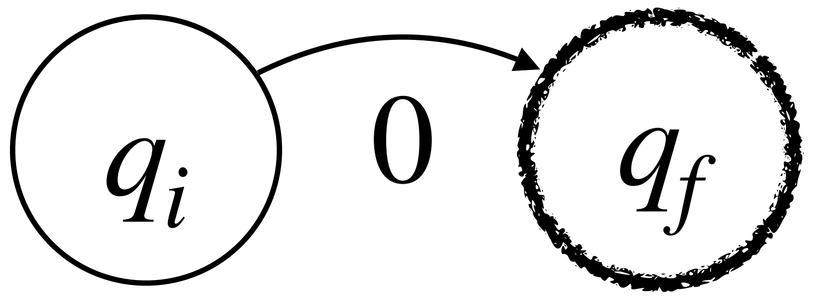
NFA from a RegEx

Example

- **Find an NFA for** $(0+1)^*(101+010)(0+1)^*$
- Rewrite:

$$\underbrace{(0+1)^*}_{N_A} \cdot \underbrace{(101+010)}_{N_B} \cdot \underbrace{(0+1)^*}_{N_A} = \underbrace{(0+1)^*}_{(N_0+N_1)^*} \cdot (\underbrace{101}_{N_C} + \underbrace{010}_{N_D}) \cdot \underbrace{(0+1)^*}_{(N_0+N_1)^*}$$

N_0



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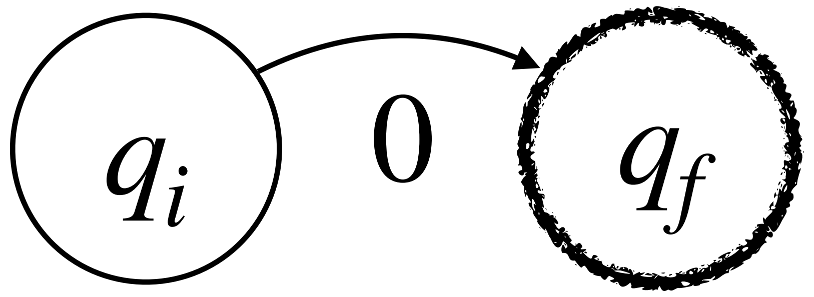
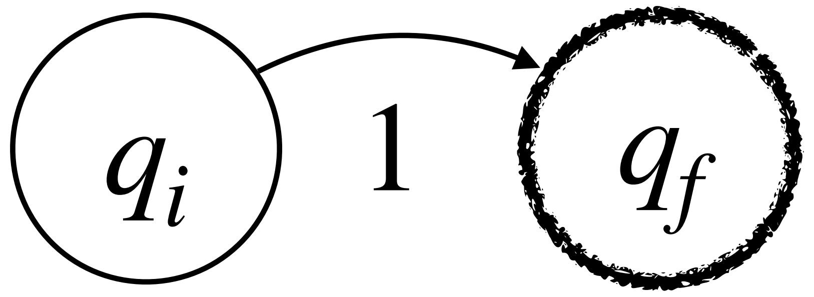
N_0



N_1



NFA from a RegEx

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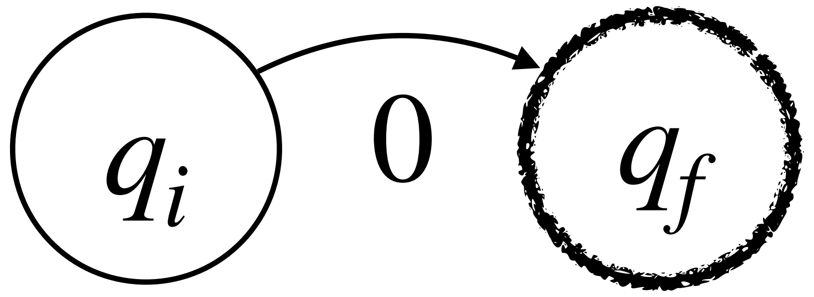
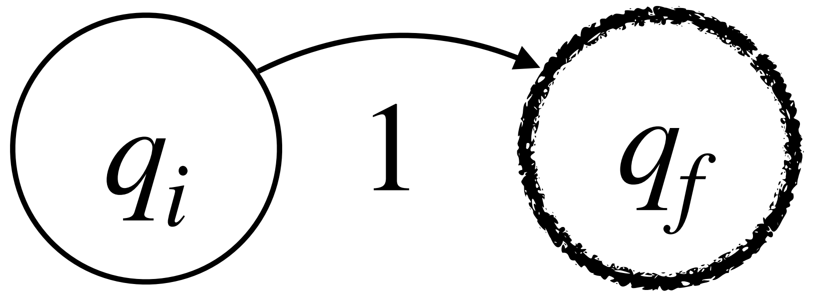
N_0



N_1



$N_0 + N_1$

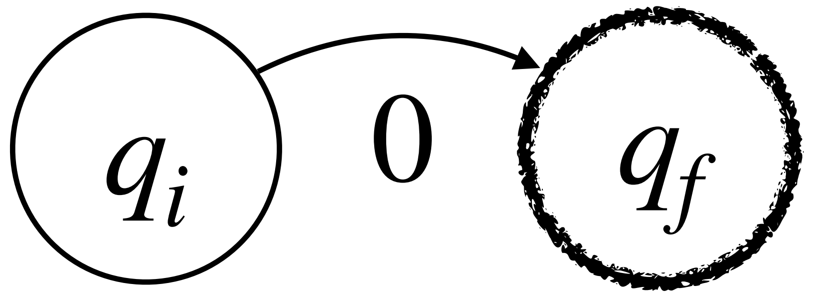
NFA from a RegEx

Example

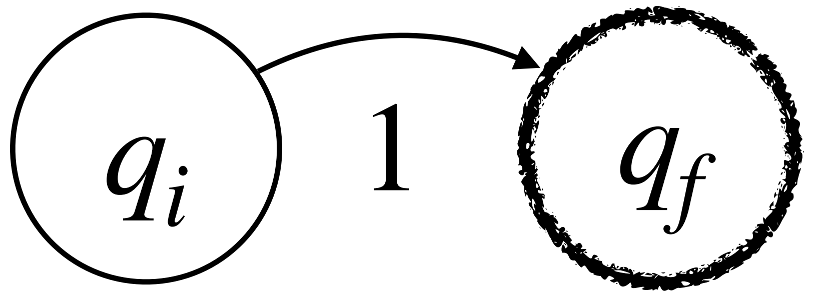
- **Find an NFA for** $(0+1)^*(101+010)(0+1)^*$
- Rewrite:

$$\underbrace{(0+1)^*}_{N_A} \cdot \underbrace{(101+010)}_{N_B} \cdot \underbrace{(0+1)^*}_{N_A} = \underbrace{(0+1)^*}_{(N_0+N_1)^*} \cdot (\underbrace{101}_{N_C} + \underbrace{010}_{N_D}) \cdot \underbrace{(0+1)^*}_{(N_0+N_1)^*}$$

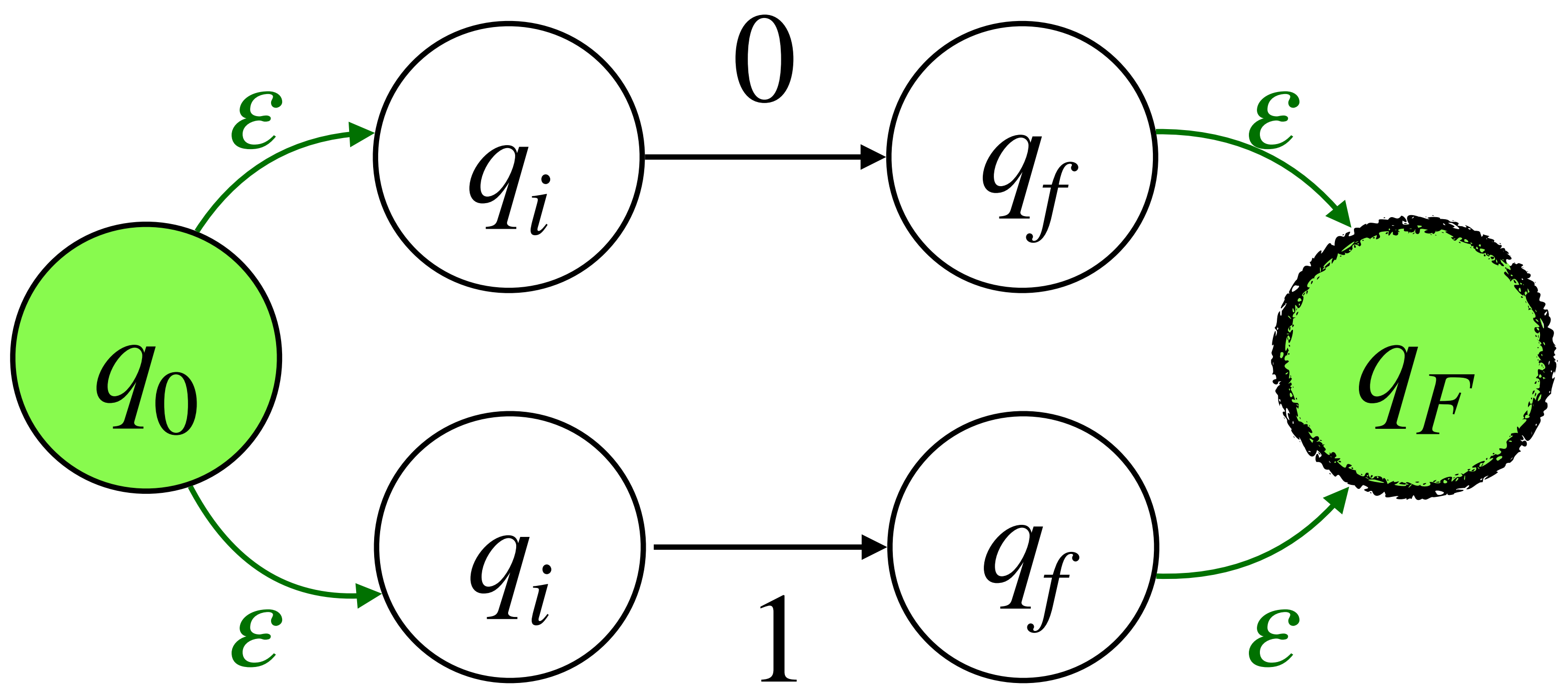
N_0



N_1



$N_0 + N_1$



NFA from a RegEx

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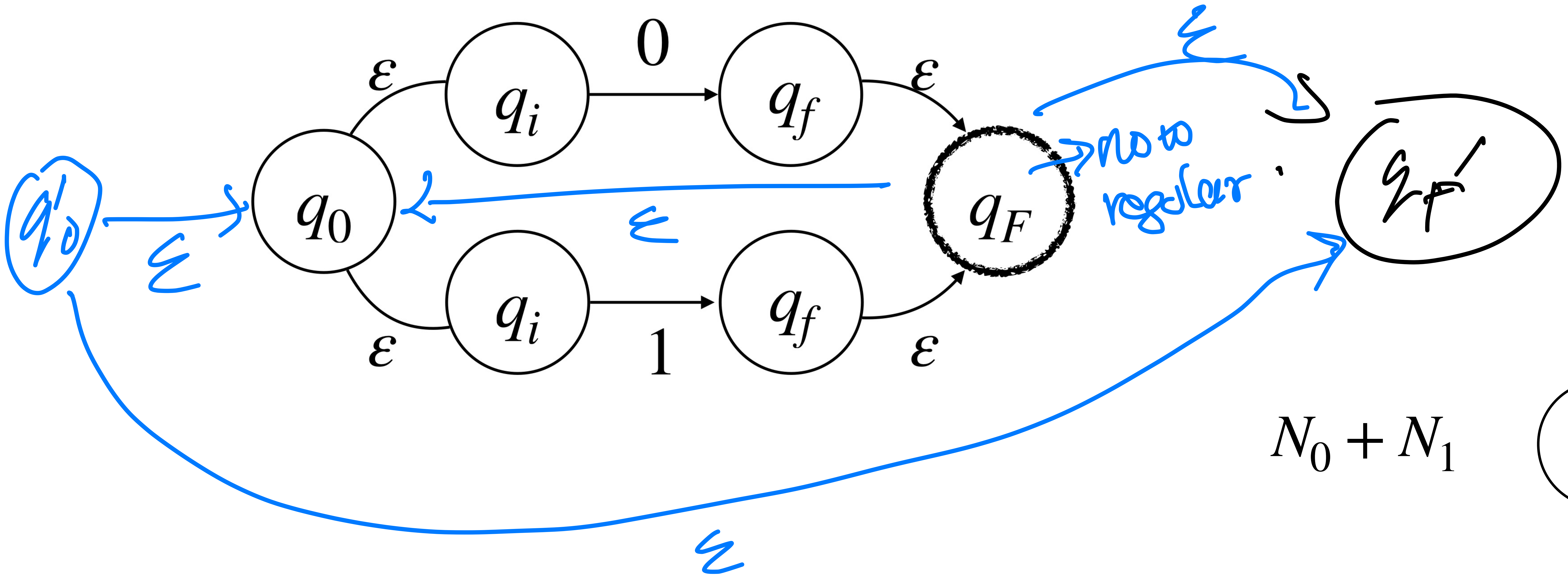
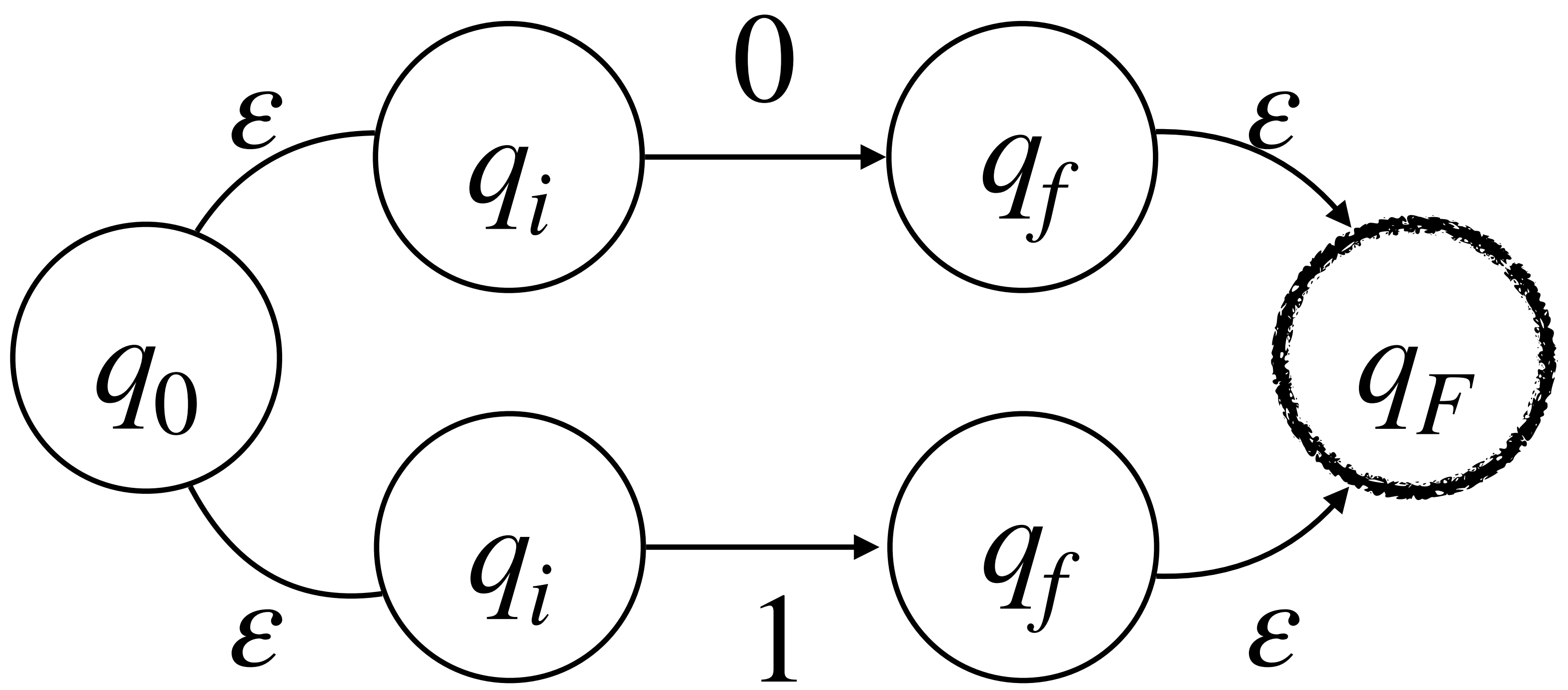


$N_0 + N_1$



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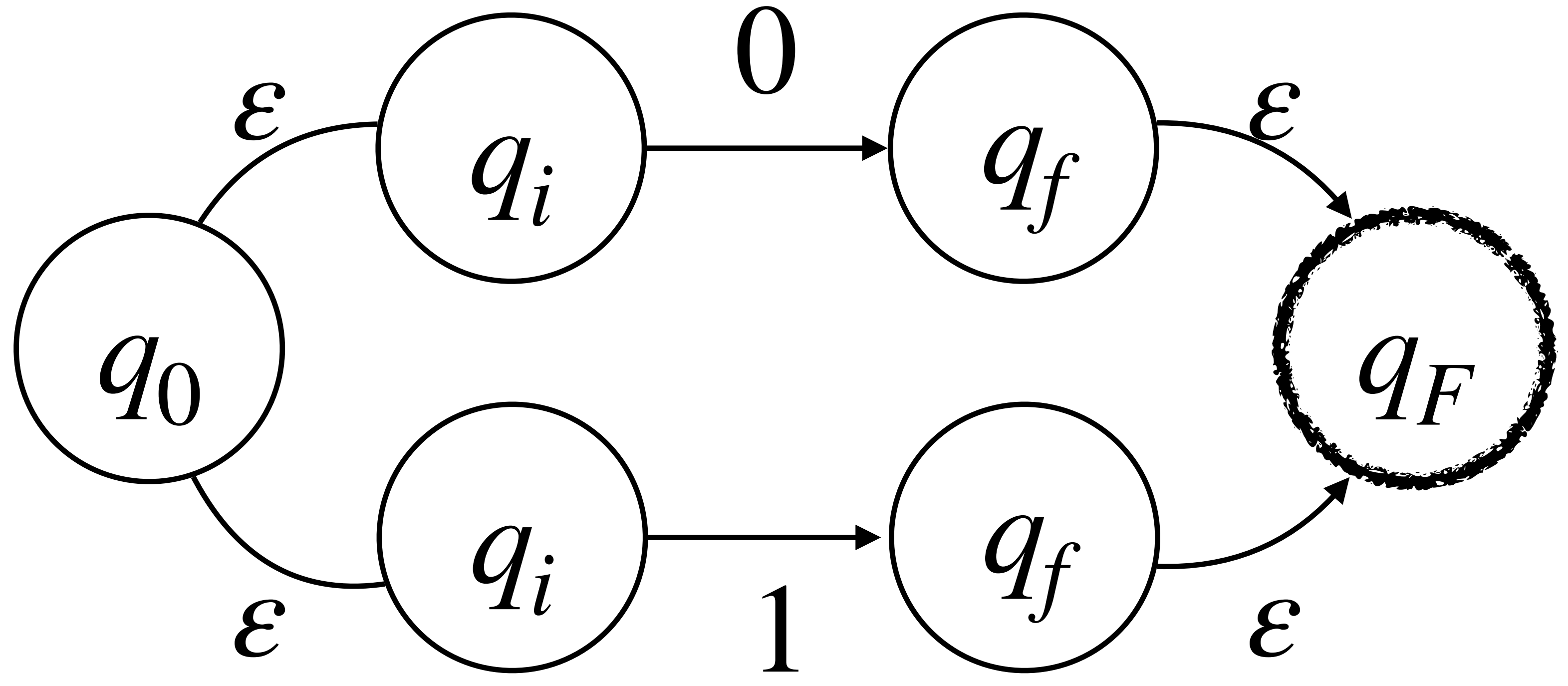
$(N_0 + N_1)^*$

$N_0 + N_1$



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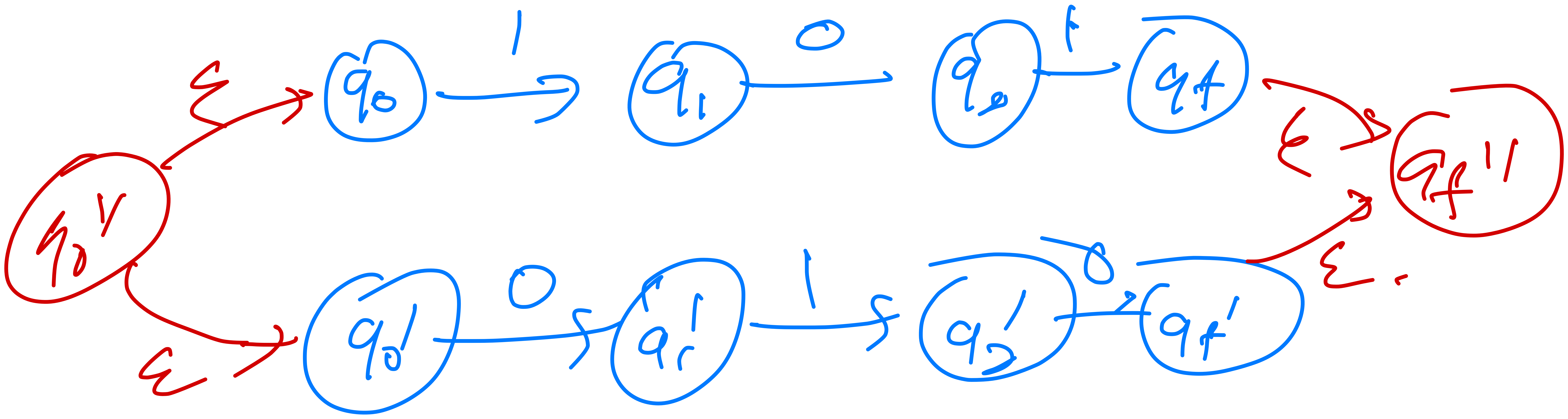
$(N_C + N_D)$



NFA from a RegEx

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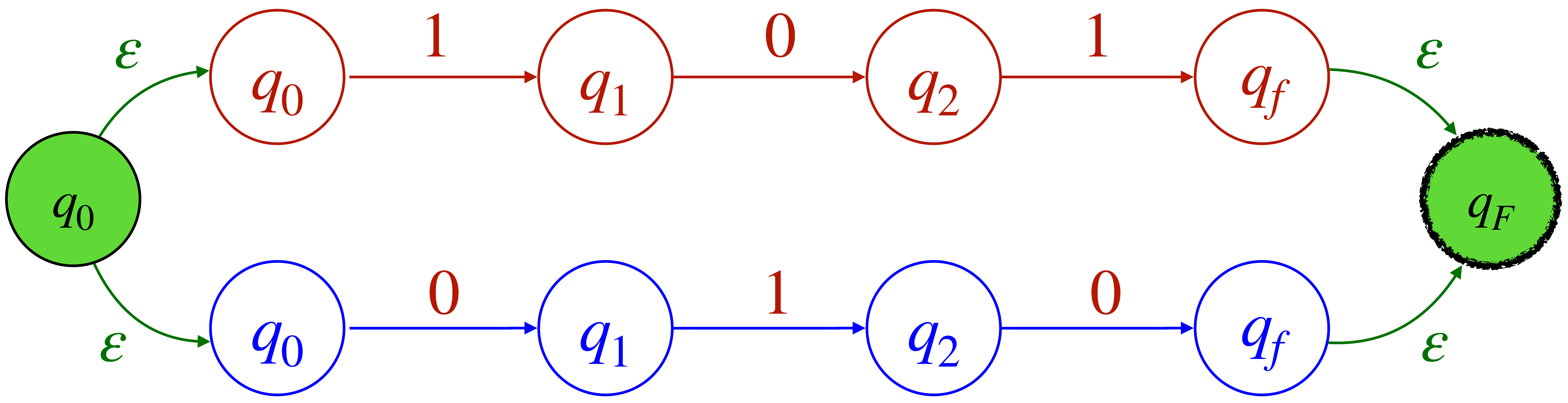
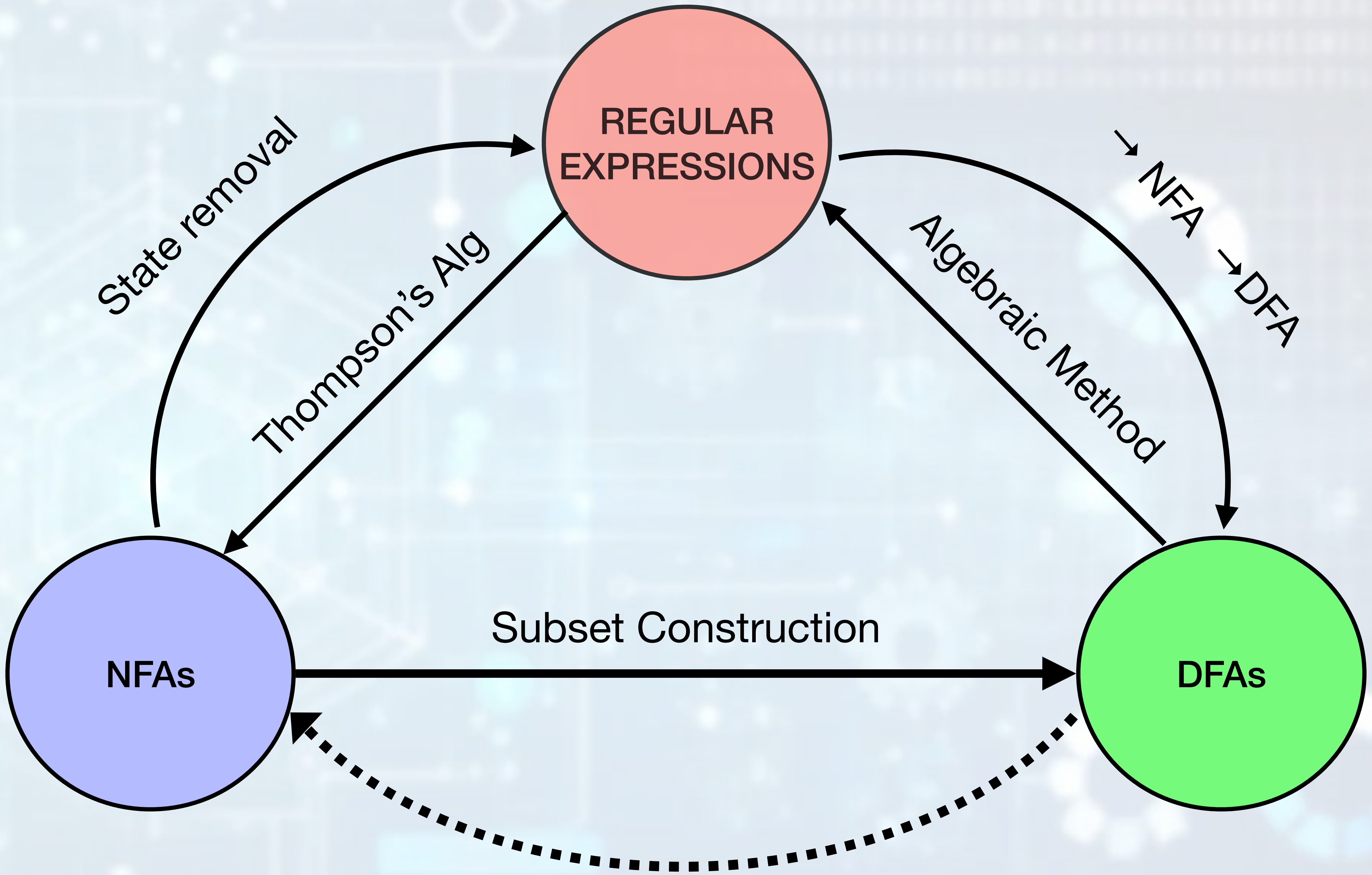


Regular Expression to DFA - Brzozowski's algorithm

Skipped - see Kani Archive for more information

Figure from Kani Archive



Summary

Next class: Languages that are not regular

Image by ChatGPT (probably collaborated with DALL-E)