ECE 374 B - Fall 2023

Homework 3

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera). We will mark difficult to read solutions as incorrect and move on.
- **Every homework problem must be done *individually*.** Each problem needs to be submitted to Gradescope before 6AM of the due data which can be found on the course website: https://ecealgo.com/homeworks.html.
- For nearly every problem, **we have covered all the requisite knowledge required to complete a homework assignment prior to the "assigned" date.** This means that there is no reason not to begin a homework assignment as soon as it is assigned. Starting a problem the night before it is due a recipe for failure.

Policies to keep in mind

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.
- **Being able to clearly and concisely explain your solution is a part of the grade you will receive.** Before submitting a solution ask yourself, if you were reading the solution without having seen it before, would you be able to understand it within two minutes? If not, you need to edit. Images and flow-charts are very useful for concisely explain difficult concepts.

See the course web site (https://ecealgo.com) for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

1. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove that the language is regular (by constructing a DFA or regular expression) or prove that the language is not regular (using fooling sets).

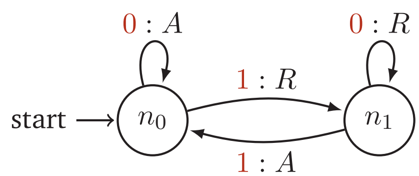
   Second, for each of these language, argue if they are context-free (or not).

   (a) $L_{1a} = \{xwwy | w, x, y \in \Sigma^+\}$

   (b) $L_{1b} = \{xww^R x^R | w, x \in \Sigma^+\}$

2. For any language A, let SkipFirstChar$(A) = \{w | aw \in A$ for some charcater $a \in \Sigma\}$. Show that the class of context-free languages is closed under the SkipFirstChar operation.

3. In a previous lab/homework we talked about a new machine called a *finite-state transducer* (FST). The special part thing about this type of machine is that it gives an output on the transition instead of the state that it is in. An example of a finite state transducer is as follows:



   defined by the five tuple: $(\Sigma, \Gamma, Q, \delta, s)$. Let's constrain this machine (call is FST_{AR}) a bit and say the output alphabet consists of two signals: accept or reject ($\Gamma = \{A, R\}$). We say that $L(FST_{AR})$ represents the language consisting of all strings that end with a accept (A) output signal.

   Prove that $L(FST_{AR})$ represents the class of regular languages.

   Alternate Problem

   An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if **every** possible state that M could be in after reading input x is a state from F. Note, this is in contrast to an ordinary NFA that accepts a string if some state among these possible states is a an accept state. Prove that all-NFAs recognize the class of regular languages.

4. Prove this language is not regular by providing a fooling set. Be sure to include the fooling set you construct is i) infinite and ii) a valid fooling set.

$$L_{P5} = \{w | w \text{ such that } |w| = \lceil k\sqrt{k} \rceil, \text{ for some natural number } k\}$$

   Hint: since this one is more difficult, we'll even give you a fooling set that works: try $F = \{0^{m^6} | m \geq 1\}$. We'll also provide a bound that can help: the difference between consecutive strings in the language, $\lceil (k+1)^{1.5} \rceil - \lceil k^{1.5} \rceil$, is bounded above and below as follows

$$1.5\sqrt{k} - 1 \leq \lceil (k+1)^{1.5} \rceil - \lceil k^{1.5} \rceil \leq 1.5\sqrt{k} + 3$$

   All that's left is you need to carefully prove that F is a fooling set for L.