

In lecture, we described an algorithm of Karatsuba that multiplies two  $n$ -digit integers using  $O(n^{\lg 3})$  single-digit additions, subtractions, and multiplications. In this lab we'll look at some extensions and applications of this algorithm.

1. Describe an algorithm to compute the product of an  $n$ -digit number and an  $m$ -digit number, where  $m < n$ , in  $O(m^{\lg 3 - 1}n)$  time. *Hint: Break up the bigger number into chunks with  $m$  bits each.*
2. Describe an algorithm to compute the decimal representation of  $2^n$  in  $O(n^{\lg 3})$  time. (The standard algorithm that computes one digit at a time requires  $\Theta(n^2)$  time.)
3. Describe a divide-and-conquer algorithm to compute the decimal representation of an arbitrary  $n$ -bit binary number in  $O(n^{\lg 3})$  time. [*Hint: Let  $x = a \cdot 2^{n/2} + b$ . Watch out for an extra log factor in the running time.*]

**Other Divide and Conquer Problems:**

4. Given an arbitrary array  $A[1..n]$ , describe an algorithm to determine in  $O(n)$  time whether  $A$  contains more than  $n/4$  copies of any value. **Do not use hashing, or radix sort, or any other method that depends on the precise input values.**

**Think about later:**

5. Suppose we can multiply two  $n$ -digit numbers in  $O(M(n))$  time. Describe an algorithm to compute the decimal representation of an arbitrary  $n$ -bit binary number in  $O(M(n)\log n)$  time.