

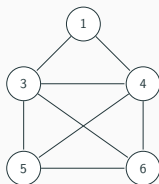
Pre-lecture brain teaser

Consider the following algorithm which takes in an undirected graph (G) and a vertex s .

```
FindClique ( $G, s$ )  
   $C = s$   
  for each vertex  $v \in V$   
     $flag = 1$   
    for each vertex  $u \in C$   
      if  $(u, v) \notin E$   
         $flag = 0$   
    if  $flag == 1$   
       $C = C \cup \{v\}$   
return  $C$ 
```

The algorithm represents a greedy algorithm which finds a clique depending on a start vertex s .

- How fast is this algorithm?



ECE-374-B: Lecture 20 - P/NP and NP-completeness

Instructor: Abhishek Kumar Umrawal

Nov 07, 2023

University of Illinois at Urbana-Champaign

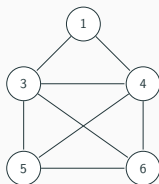
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The algorithm is a represents a greedy algorithm which finds a clique depending on a start vertex s .

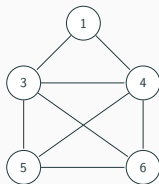
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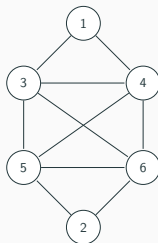


The Clique-problem is NP-complete. But this algorithm provides us with the maximal clique containing s . If we run it $|V|$ times, does that solve the clique-problem.

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         $flag = 0$   
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  return  $C$ 
```



The Satisfiability Problem (SAT)

Propositional Formulas

Definition

Consider a set of boolean variables x_1, x_2, \dots, x_n .

- A literal is either a boolean variable x_i or its negation $\neg x_i$.
- A clause is a disjunction of literals.
For example, $x_1 \vee x_2 \vee \neg x_4$ is a clause.
- A formula in conjunctive normal form (CNF) is propositional formula which is a conjunction of clauses.
 - $(x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee \neg x_3) \wedge x_5$ is a CNF formula.

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 - $(x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee \neg x_3) \wedge x_5$ is a **CNF** formula.
- A formula φ is a **3CNF**:
A **CNF** formula such that every clause has **exactly** 3 literals.
 - $(x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee \neg x_3 \vee x_1)$ is a **3CNF** formula, but
 $(x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee \neg x_3) \wedge x_5$ is not.

CNF is universal

Every boolean formula $f : \{0, 1\}^n \rightarrow \{0, 1\}$ can be written as a CNF formula.

x_1	x_2	x_3	x_4	x_5	x_6	$f(x_1, x_2, \dots, x_6)$	$\overline{x_1} \vee x_2 \overline{x_3} \vee x_4 \vee \overline{x_5} \vee x_6$
0	0	0	0	0	0	$f(0, \dots, 0, 0)$	1
0	0	0	0	0	1	$f(0, \dots, 0, 1)$	1
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
1	0	1	0	0	1	?	1
1	0	1	0	1	0	0	0
1	0	1	0	1	1	?	1
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	
1	1	1	1	1	1	$f(1, \dots, 1)$	1

How? For every row such that f is zero, compute corresponding CNF clause. Then take the AND (\wedge) of all the CNF clauses computed. The resulting CNF formula is equivalent to f .

Problem: SAT

Instance: A CNF formula φ .

Question: Is there a truth assignment to the variable of φ such that φ evaluates to true?

Problem: 3SAT

Instance: A 3CNF formula φ .

Question: Is there a truth assignment to the variable of φ such that φ evaluates to true?

Satisfiability

SAT

Given a **CNF** formula φ , is there a truth assignment to variables such that φ evaluates to true?

Example

- $(x_1 \vee x_2 \vee \neg x_4) \wedge (x_2 \vee \neg x_3) \wedge x_5$ is satisfiable; take x_1, x_2, \dots, x_5 to be all true
- $(x_1 \vee \neg x_2) \wedge (\neg x_1 \vee x_2) \wedge (\neg x_1 \vee \neg x_2) \wedge (x_1 \vee x_2)$ is not satisfiable.

3SAT

Given a **3CNF** formula φ , is there a truth assignment to variables such that φ evaluates to true?

Importance of **SAT** and **3SAT**

- **SAT** and **3SAT** are basic constraint satisfaction problems.
- Many different problems can be reduced to them because of the simple yet powerful expressiveness of logical constraints.
- Arise naturally in many applications involving hardware and software verification and correctness.
- As we will see, it is a fundamental problem in theory of NP-Completeness.

$$z = \bar{x}$$

Given two bits x, z which of the following **SAT** formulas is equivalent to the formula $z = \bar{x}$:

(A) $(\bar{z} \vee x) \wedge (z \vee \bar{x})$.

(B) $(z \vee x) \wedge (\bar{z} \vee \bar{x})$.

(C) $(\bar{z} \vee x) \wedge (\bar{z} \vee \bar{x}) \wedge (\bar{z} \vee \bar{x})$.

(D) $z \oplus x$.

(E) $(z \vee x) \wedge (\bar{z} \vee \bar{x}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x)$.

Answer: B

$z = \bar{x}$: Solution

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- (C) $(\bar{z} \vee x) \wedge (\bar{z} \vee \bar{x}) \wedge (\bar{z} \vee \bar{x})$.
- (D) $z \oplus x$.
- (E) $(z \vee x) \wedge (\bar{z} \vee \bar{x}) \wedge (z \vee \bar{x}) \wedge (\bar{z} \vee x)$.

x	y	$z = \bar{x}$
0	0	0
0	1	1
1	0	1
1	1	0

$$z = x \wedge y$$

Given three bits x, y, z which of the following **SAT** formulas is equivalent to the formula $z = x \wedge y$:

(A) $(\bar{z} \vee x \vee y) \wedge (z \vee \bar{x} \vee \bar{y})$.

(B) $(\bar{z} \vee x \vee y) \wedge (\bar{z} \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y})$.

(C) $(\bar{z} \vee x \vee y) \wedge (\bar{z} \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y})$.

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(E) $(z \vee x \vee y) \wedge (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge$
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Answer: C

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- (E) $(z \vee x \vee y) \wedge (z \vee x \vee \bar{y}) \wedge (z \vee \bar{x} \vee y) \wedge (z \vee \bar{x} \vee \bar{y}) \wedge (\bar{z} \vee x \vee y) \wedge (\bar{z} \vee x \vee \bar{y}) \wedge$

x	y	z	$z = x \wedge y$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Reducing SAT to 3SAT

How **SAT** is different from **3SAT**?

In **SAT** clauses might have arbitrary length: 1, 2, 3, ... variables:

$$(x \vee y \vee z \vee w \vee u) \wedge (\neg x \vee \neg y \vee \neg z \vee w \vee u) \wedge (\neg x)$$

In **3SAT** every clause must have exactly 3 different literals.

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In **3SAT** every clause must have exactly 3 different literals.

To reduce from an instance of **SAT** to an instance of **3SAT**, we must make all clauses to have exactly 3 variables...

Basic idea

- Pad short clauses so they have 3 literals.
- Break long clauses into shorter clauses.
- Repeat the above till we have a **3CNF**.

Proof of this in Prof. Har-Peled's async lectures!

Overview of Complexity Classes

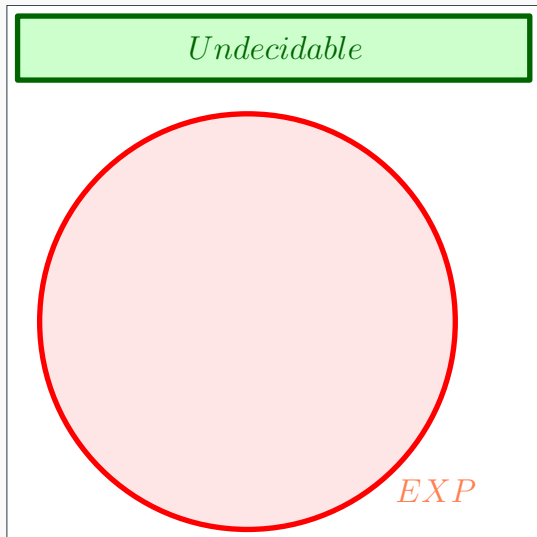
In the beginning...



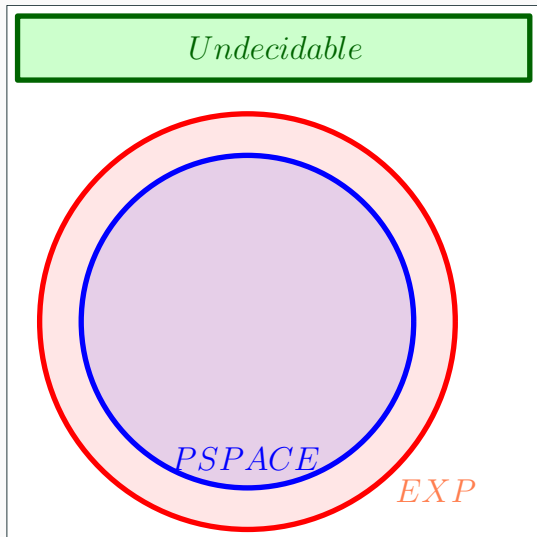
In the beginning...

Undecidable

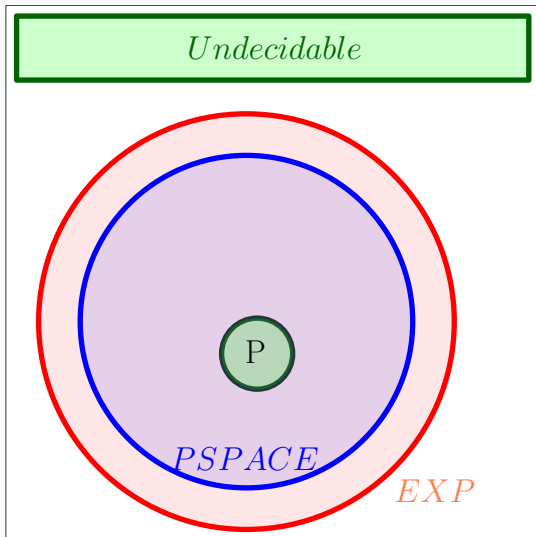
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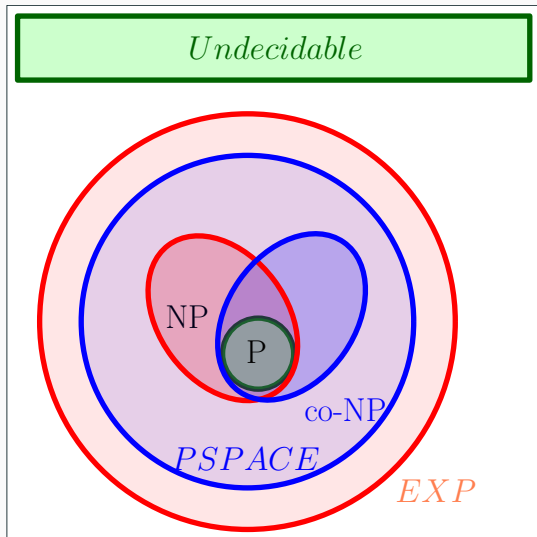
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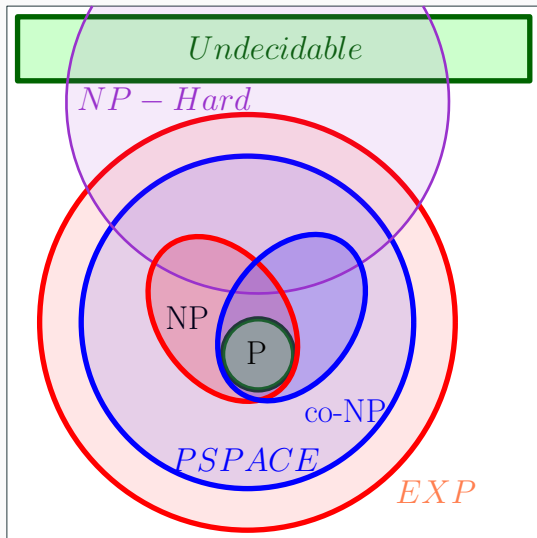
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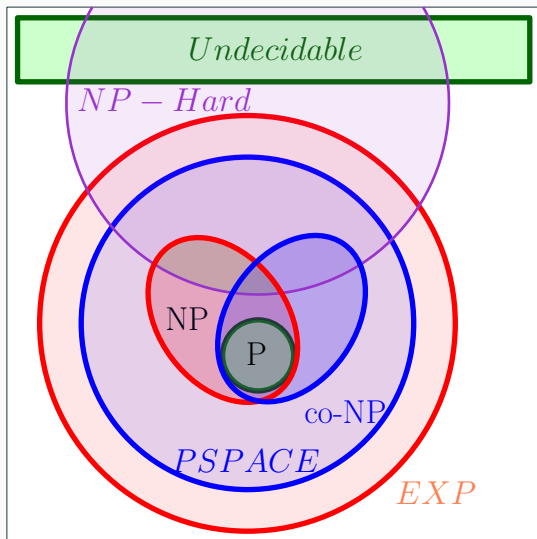
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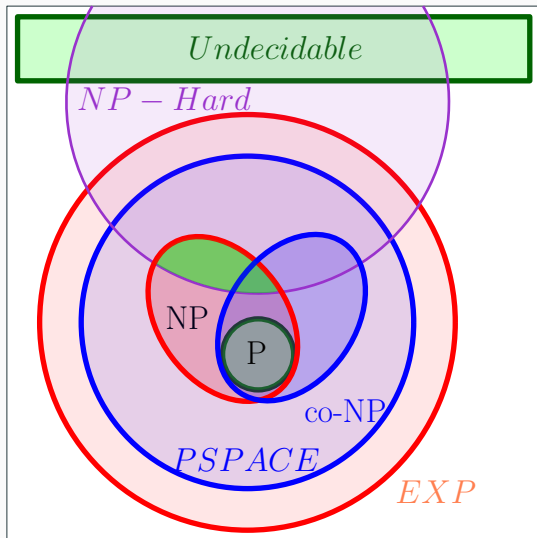
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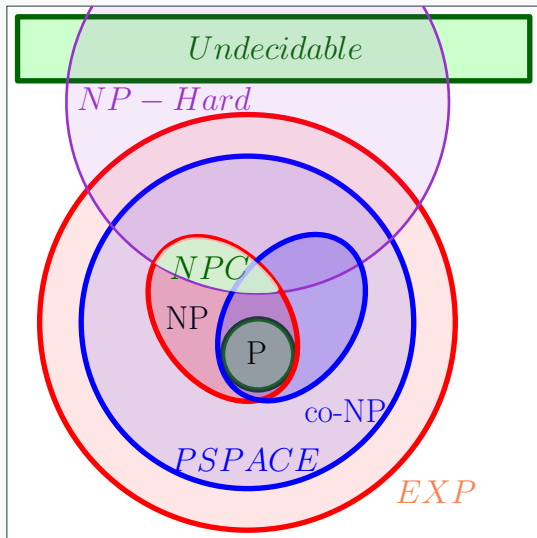
In the beginning...



In the beginning...



In the beginning...



Non-deterministic polynomial time - NP

P, NP and Turing Machines

- P : set of decision problems that have polynomial time (deterministic) algorithms, i.e. efficiently solvable using a (deterministic) Turing machine (DTM).
- NP : set of decision problems that have polynomial time non-deterministic algorithms, i.e. efficiently solvable using a non-deterministic Turing machine (NTM).
- Many natural problems we would like to solve are in NP .
- Every problem in NP has an exponential time (deterministic) algorithm.
- $P \subseteq NP$.
- Some problems in NP are in P (e.g., shortest path problem).

Big Question: Does every problem in NP have an efficient algorithm? Same as asking whether $P = NP$.

Problems with no known deterministic polynomial time algorithms

Problems

- **Independent Set**
- **Vertex Cover**
- **Set Cover**
- **SAT**

There are of course undecidable problems (no algorithm at all!) but many problems that we want to solve are of similar flavor to the above.

Question: What is common to above problems?

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Question: What is common to above problems?

They can all be solved via a non-deterministic computer in polynomial time!

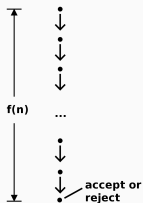
Non-determinism in computing

Non-determinism is a special property of algorithms.

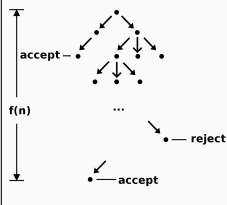
An algorithm that is capable of taking multiple states concurrently. Whenever it reaches a choice, it takes both paths.

If there is a path for the string to be accepted by the machine, then the string is part of the language.

Deterministic



Non-Deterministic



Problems with no known deterministic polynomial time algorithms

Problems

- **Independent Set** & **Vertex Cover** - Can build algorithm to check all possible collection of vertices
- **Set Cover** - Can check all possible collection of sets
- **SAT** -Can build a non-deterministic algorithm that checks every possible boolean assignment.

But we don't have access to a non-deterministic computer. So how can a deterministic computer verify that a algorithm is in NP?

Above problems share the following feature.

Checkability

For any YES instance I_X of X there is a proof/certificate/solution that is of length $\text{poly}(|I_X|)$ such that given a proof one can efficiently check that I_X is indeed a YES instance.

Efficient Checkability

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Checkability

For any YES instance I_X of X there is a proof/certificate/solution that is of length $\text{poly}(|I_X|)$ such that given a proof one can efficiently check that I_X is indeed a YES instance.

Examples:

- **SAT** formula φ : proof is a satisfying assignment.
- **Independent Set** in graph G and k : a subset S of vertices.
- **Homework**.

Definition

An algorithm $C(\cdot, \cdot)$ is a certifier for problem X if the following two conditions hold.

- For every $s \in X$ there is some string t such that $C(s, t) = \text{"yes"}$
- If $s \notin X$, $C(s, t) = \text{"no"}$ for every t .

The string s is the problem instance. (Example: particular graph in independent set problem.) The string t is called a **certificate** or **proof** for s .

Efficient (polynomial time) Certifiers

Definition (Efficient Certifier.)

A certifier C is an efficient certifier for problem X if there is a polynomial $p(\cdot)$ such that the following conditions hold.

- For every $s \in X$ there is some string t such that $C(s, t) = \text{"yes"}$ **and** $|t| \leq p(|s|)$.
- If $s \notin X$, $C(s, t) = \text{"no"}$ for every t .
- $C(\cdot, \cdot)$ runs in polynomial time.

Example: Independent Set

- **Problem:** Does $G = (V, E)$ have an independent set of size $\geq k$?
 - **Certificate:** Set $S \subseteq V$.
 - **Certifier:** Check $|S| \geq k$ and no pair of vertices in S is connected by an edge.

Example: SAT

- **Problem:** Does formula φ have a satisfying truth assignment?
 - **Certificate:** Assignment a of 0/1 values to each variable.
 - **Certifier:** Check each clause under a and say “yes” if all clauses are true.

Why is it called Non-deterministic Polynomial Time

A certifier is an algorithm $C(I, c)$ with the following two inputs.

- I : instance.
- c : proof/certificate that the instance is indeed a YES instance of the given problem.

One can think about C as an algorithm for the original problem if the following hold.

- Given I , the algorithm guesses (non-deterministically, and who knows how) a certificate c .
- The algorithm now verifies the certificate c for the instance I .

NP can be equivalently described using Turing machines.

Cook-Levin Theorem

“Hardest” Problems

Question

What is the hardest problem in NP? How do we define it?

Towards a definition

- Hardest problem must be in NP.
- Hardest problem must be at least as “difficult” as every other problem in NP.

NP-Complete Problems

Definition

A problem X is said to be **NP-Complete** if

- $X \in NP$, and
- (**Hardness**) For any $Y \in NP$, $Y \leq_P X$.

Solving NP-Complete Problems

Lemma

Suppose X is NP-Complete. Then X can be solved in polynomial time if and only if $P = NP$.

Proof.

⇒ Suppose X can be solved in polynomial time

- Let $Y \in NP$. We know $Y \leq_P X$.
- We showed that if $Y \leq_P X$ and X can be solved in polynomial time, then Y can be solved in polynomial time.
- Thus, every problem $Y \in NP$ is such that $Y \in P$; $NP \subseteq P$.
- Since $P \subseteq NP$, we have $P = NP$.

⇐ Since $P = NP$, and $X \in NP$, we have a polynomial time algorithm for X . □

NP-Hard Problems

Definition

A problem Y is said to be **NP-Hard** if

- (**Hardness**) For any $X \in NP$, we have that $X \leq_P Y$.

An NP-Hard problem need not be in NP!

Example: Halting problem is NP-Hard (why?) but not NP-Complete.

Consequences of proving NP-Completeness

If X is NP-Complete

- Since we believe $P \neq NP$,
- and solving X implies $P = NP$.

X is **unlikely** to be efficiently solvable.

At the very least, many smart people before you have failed to find an efficient algorithm for X .

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(This is proof by mob opinion — take with a grain of salt.)

NP-Complete Problems

Question

Are there any problems that are NP-Complete?

Answer

Yes! Many, many problems are NP-Complete.

Cook-Levin Theorem

Theorem (Cook-Levin)
SAT is *NP-Complete*.

Cook-Levin Theorem

Theorem (Cook-Levin)

SAT is NP-Complete.

Need to show the following.

- **SAT** is in NP.
- Every NP problem X reduces to **SAT**.

Steve Cook won the Turing award for his theorem.

Proving that a problem X is NP-Complete

To prove X is NP-Complete, show the following.

- Show that X is in NP.
- Give a polynomial-time reduction from a known NP-Complete problem such as **SAT** to X .

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SAT \leq_P X implies that every NP problem $Y \leq_P X$. Why?

Transitivity of reductions:

$Y \leq_P \text{SAT}$ and $\text{SAT} \leq_P X$ and hence $Y \leq_P X$.

3-SAT is NP-Complete

- **3-SAT** is in *NP*.
- **SAT** \leq_P **3-SAT** as we saw.

NP-Completeness via Reductions

- **SAT** is NP-Complete due to Cook-Levin theorem.
- **SAT** \leq_P **3-SAT**
- **3-SAT** \leq_P **Independent Set**
- **Independent Set** \leq_P **Vertex Cover**
- **Independent Set** \leq_P **Clique**
- **3-SAT** \leq_P **3-Color**
- **3-SAT** \leq_P **Hamiltonian Cycle**

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Hundreds and thousands of different problems from many areas of science and engineering have been shown to be NP-Complete.

A surprisingly frequent phenomenon!

Reducing 3-SAT to Independent Set

Independent Set

Problem: Independent Set

Instance: A graph G , integer k .

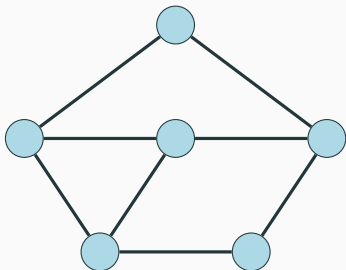
Question: Is there an independent set in G of size k ?

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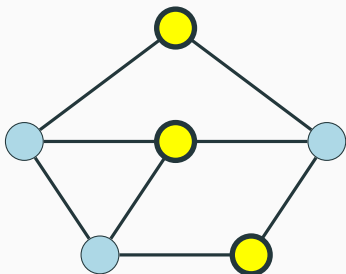


Independent Set

Problem: Independent Set

Instance: A graph G , integer k .

Question: Is there an independent set in G of size k ?



There are two ways to think about **3SAT**.

1. Find a way to assign 0/1 (false/true) to the variables such that the formula evaluates to true, that is each clause evaluates to true.
2. Pick a literal from each clause and find a truth assignment to make all of them true. You will fail if two of the literals you pick are in **conflict**, i.e., you pick x_i and $\neg x_i$.

We will take the second view of **3SAT** to construct the reduction.

The Reduction

1. G_φ will have one vertex for each literal in a clause.
2. Connect the literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true.
3. Connect 2 vertices if they label complementary literals; this ensures that the literals corresponding to the independent set do not have a conflict.
4. Take k to be the number of clauses.

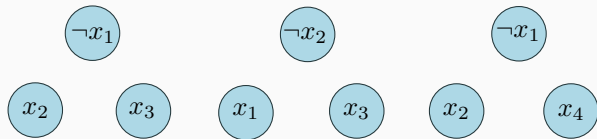


Figure 1: Graph for

$$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4).$$

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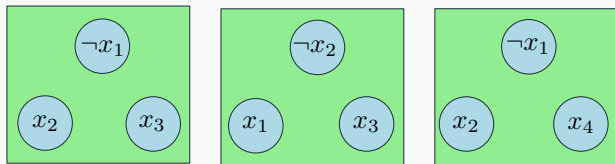


Figure 1: Graph for

$$\varphi = (\neg x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \neg x_2 \vee x_3) \wedge (\neg x_1 \vee x_2 \vee x_4).$$

The Reduction

1. G_φ will have one vertex for each literal in a clause.
2. Connect the literals in a clause to form a triangle; the independent set will pick at most one vertex from each clause, which will correspond to the literal to be set to true.
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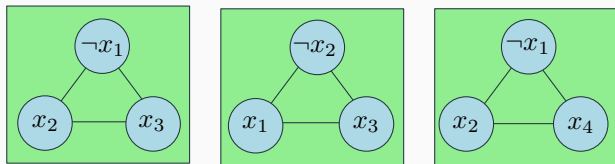


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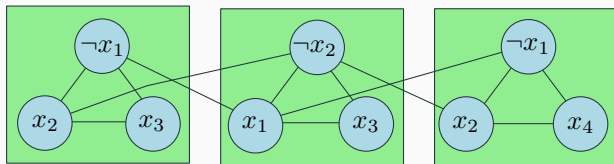


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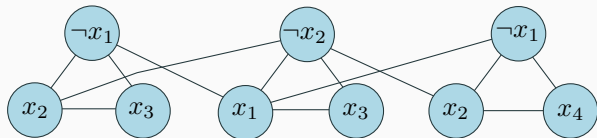


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Lemma

φ is satisfiable iff G_φ has an independent set of size k (= number of clauses in φ).

Proof.

\Rightarrow Let a be the truth assignment satisfying φ .

- Pick one of the vertices, corresponding to true literals under a , from each triangle. This is an independent set of the appropriate size. Why? □

Correctness (contd)

Lemma

φ is satisfiable iff G_φ has an independent set of size k (= number of clauses in φ).

Proof.

\Leftarrow Let S be an independent set of size k .

- S must contain exactly one vertex from each clause triangle.
- S cannot contain vertices labeled by conflicting literals.
- Thus, it is possible to obtain a truth assignment that makes in the literals in S true; such an assignment satisfies one literal in every clause. \square

Other NP-Complete problems

Graph Coloring

Problem: Graph Coloring

Instance: $G = (V, E)$: Undirected graph, integer k .

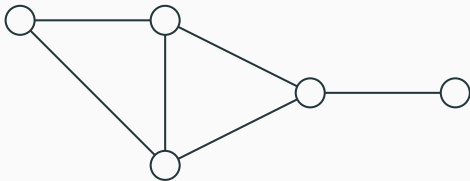
Question: Can the vertices of the graph be colored using k colors so that vertices connected by an edge do not get the same color?

Graph 3-Coloring

Problem: 3 Coloring

Instance: $G = (V, E)$: Undirected graph.

Question: Can the vertices of the graph be colored using 3 colors so that vertices connected by an edge do not get the same color?

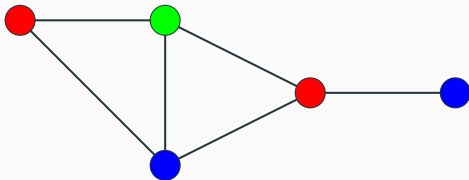


Graph 3-Coloring

Problem: 3 Coloring

Instance: $G = (V, E)$: Undirected graph.

Question: Can the vertices of the graph be colored using 3 colors so that vertices connected by an edge do not get the same color?



Graph Coloring

Observation: If G is colored with k colors then each color class (nodes of same color) form an independent set in G . Thus, G can be partitioned into k independent sets iff G is k -colorable.

Graph 2-Coloring can be decided in polynomial time.

G is 2-colorable iff G is bipartite! There is a linear time algorithm to check if G is bipartite using breadth first search.

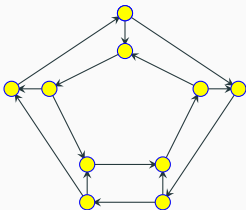
Hamiltonian Cycle

Directed Hamiltonian Cycle

Input Given a directed graph $G = (V, E)$ with n vertices

Goal Does G have a **Hamiltonian cycle**?

- A Hamiltonian cycle is a cycle in the graph that visits every vertex in G exactly once.



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