Problem type 1:

Provide a infinite fooling set for the following problem:

(See variants below)

You must show why it is a fooling set (mainly by showing that for any two elements in the fooling set, there is a suffix that makes them distinguishable). Assume $\Sigma = (\mathtt{0}, \mathtt{1})$

a. **BYB**

$L = \{\mathtt{0}^a \mathtt{1}^a \mathtt{0}^a | a \geq 0\}$

Solution: Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i | i \geq 0\right\}$. For any two strings in this language: $x = \mathtt{0}^i$, $y = \mathtt{0}^j$, (where $i \neq j$) there exists a suffix $z = \mathtt{1}^i\mathtt{0}^i$, for which $xz \in L$ and $yz \notin L$. ■

b. **BYA**

$L = \left\{\mathtt{0}^a \mathtt{1}\mathtt{0}^b | 0 \leq a \leq b\right\}$

Solution: Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i | i \geq 0\right\}$. For any two strings in this language: $x = \mathtt{0}^i$, $y = \mathtt{0}^j$ (where $i \leq j$), there exists a suffix $z = \mathtt{1}\mathtt{0}^i$, for which $xz \in L$ and $yz \notin L$. ■

c. **BYC**

$L = \left\{\mathtt{0}^a \mathtt{1}^b \mathtt{0}^c \mathtt{1}^d | a + c = b + d\right\}$

Solution: Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i \mathtt{1}^i \mathtt{0}^i | i \geq 0\right\}$. For any two strings in this language: $x = \mathtt{0}^i\mathtt{1}^i\mathtt{0}^i$, $y = \mathtt{0}^j\mathtt{1}^j\mathtt{0}^j$ (where $i \neq j$), there exists a suffix $z = \mathtt{1}^i$, for which $xz \in L$ and $yz \notin L$. ■

d. **BYG**

$L = \{w\mathtt{1}w | w \in \Sigma^*\}$

Solution: Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i\right\}$. For any two strings in this language: $x = \mathtt{0}^i$, $y = \mathtt{0}^j$ (where $i \neq j$), there exists a suffix $z = \mathtt{1}\mathtt{0}^i$, for which $xz \in L$ and $yz \notin L$. ■

Problem type 2:

For the following language, please state whether it is *regular*, or ***not** regular*.

> *(See variants below)*

If the language is regular prove it by writing down a regex/DFA/NFA that represents the language. If the language is not regular, provide a infinite fooling set. You must show why it is a fooling set (mainly by showing that for any two elements in the fooling set, there is a suffix that makes them distinguishable). Assume $\Sigma = (\mathtt{0}, \mathtt{1})$.

a. **BYF**

$L = \left\{\mathtt{0}^a\mathtt{0}^b | 0 \le a \le b\right\}$

Solution: Well despite how it is written, the two runs of $\mathtt{0}$'s can combine. This language is effectively all runs of $\mathtt{0}$'s and can be represented by the regex $r = \mathtt{0}^*$ ∎

b. **BYH**

$L = \{xw | x, w \in \Sigma^* \text{ and } x \neq w\}$

Solution: This one is very similar to the Part 2 questions in Lab 5. In this case, it is tempting to go with not regular because you have two strings and a conditional between them. But wait! These two strings are right next to eachother and whatever string is inputted into L, let's say the input is $\mathtt{0101}$, this can be part of the language if $x = \mathtt{0101}$ and $w = \varepsilon$. Using this logic, the only string that isn't accepted by L is ε because in that case, both x and w would have to be equal to the empty string.

Knowing this, we can formulate the regex that describes this language as: $(\mathtt{0} + \mathtt{1})^+$. ∎

c. **BYE**

$L = \{w\mathtt{1}x | w, x \in \Sigma^* \text{ and } w \neq x\}$

Solution: This one is very close to the prior question, but in this case, we have a delimiter separating x and w and that pesky $\mathtt{1}$ makes all this difference and makes this a non regular langauge.

Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i\right\}$. For any two strings in this language: $x = \mathtt{0}^i$, $y = \mathtt{0}^j$ (where $i \neq j$), there exists a suffix $z = \mathtt{10}^i$, for which $xz \notin L$ and $yz \in L$. ∎

d. **BYD**

$L = \left\{\mathtt{0}^a\mathtt{10}^b | 0 \le a \le b\right\}$

Solution: This one is very close to the prior question, but in this case, we have a delimiter separating $\mathtt{0}^a$ and $\mathtt{0}^b$ and that pesky $\mathtt{1}$ makes all this difference and makes this a non regular langauge.

Multiple correct answers, but one possibe one is: $F = \left\{\mathtt{0}^i\right\}$. For any two strings in this language: $x = \mathtt{0}^i$, $y = \mathtt{0}^j$ (where $i < j$), there exists a suffix $z = \mathtt{10}^i$, for which

$xz \notin L$ and $yz \in L$. ■