Problem type 1:

Provide the context-free grammar that describes the following language:

(See variants below)

a. **BYH**

$L = \left\{(\mathtt{0} + \mathtt{1})^*\right\}$ (all strings) where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: $S \to \varepsilon \mid \mathtt{0}S \mid \mathtt{1}S$. ■

b. **BYE**

$L = \{\mathtt{0}^n\mathtt{1}\mathtt{0}^n | n \geq 0\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: $S \to \mid \mathtt{0}S\mathtt{0} \mid \mathtt{1}$. ■

c. **BYA**

$L = \{\mathtt{0}^n\mathtt{1}^n | n \geq 0\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: $S \to \varepsilon \mid \mathtt{0}S\mathtt{1}$. ■

d. **BYB**

$L = \{\mathtt{0}^m\mathtt{1}^n | m \leq n\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: $S \to \varepsilon \mid \mathtt{0}S\mathtt{1} \mid S\mathtt{1}$. ■

e. **BYF**

$L = \{\mathtt{0}^m\mathtt{1}^n | m \neq n\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: We either have more 0's than 1's or more 1's than 0's.

$$\begin{aligned} S &\to \mathtt{0}S\mathtt{1} \mid A \mid B \\ A &\to \mathtt{0} \mid \mathtt{0}A \\ B &\to \mathtt{1} \mid B\mathtt{1} \end{aligned}$$

■

f. **BYG**

$L = \left\{\mathtt{0}^a\mathtt{1}^b\mathtt{2}^c | a, b, c \geq 0, a + b = c\right\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}, \mathtt{2}\}$

Solution:

$$\begin{aligned} S &\to \varepsilon \mid \mathtt{0}S\mathtt{2} \mid A \\ A &\to \varepsilon \mid \mathtt{1}A\mathtt{2} \end{aligned}$$

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g. **BYD**

$L = \left\{ \mathtt{0}^a \mathtt{1}^b \mathtt{2}^c | a, b, c \geq 0, a + b \leq c \right\}$ where $\Sigma = \{\mathtt{0}, \mathtt{1}, \mathtt{2}\}$

Solution:

$$S \rightarrow \varepsilon \mid \mathtt{0}S\mathtt{2} \mid A$$
$$A \rightarrow \varepsilon \mid \mathtt{1}A\mathtt{2} \mid A\mathtt{2}$$

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h. **BYC**

$L = \left\{ ww^R | w \in \Sigma^* \right\}$ (all even length palindromes) where $\Sigma = \{\mathtt{0}, \mathtt{1}\}$

Solution: $S \rightarrow \varepsilon \mid \mathtt{0}S\mathtt{0} \mid \mathtt{1}S\mathtt{1}$ ∎