Problem type 1:

In the last two lectures, we exhaustively went over the DETERMINISTIC TIME SELECTION(DTS) (aka Median of medians, Quick Select with MoM, etc.).

In the algorithm we break the input array into lists of size 5 and then get the median of each of the lists, then choose the median of those medians as the answer. In the pre-lecture yesterday, we talked about the reasons why lists of size 5 were chosen, but we can choose lists of any size and the algorithm would still run in $O(n)$ time... right?

Imagine instead of lists of size 5, we break the input array into lists of size x (given below). What is the recurrence that best describes this new DTS algorithm assuming lists of size x.

(See variants below)

a. **BYG**

$x = 9$

Solution: $T(n) = T\left(\frac{13}{18}n\right) + T\left(\frac{1}{9}n\right) + O(n)$ ■

b. **BYE**

$x = 13$

Solution: $T(n) = T\left(\frac{19}{26}n\right) + T\left(\frac{1}{13}n\right) + O(n)$ ■

c. **BYC**

$x = 15$

Solution: $T(n) = T\left(\frac{22}{30}n\right) + T\left(\frac{1}{15}n\right) + O(n)$ ■

d. **BYH**

$x = 21$

Solution: $T(n) = T\left(\frac{31}{42}n\right) + T\left(\frac{1}{21}n\right) + O(n)$ ■

e. **BYB**

$x = 23$

Solution: $T(n) = T\left(\frac{34}{46}n\right) + T\left(\frac{1}{23}n\right) + O(n)$ ■

f. **BYA**

$x = 31$

Solution: $T(n) = T\left(\frac{46}{62}n\right) + T\left(\frac{1}{31}n\right) + O(n)$ ■

g. **BYF**

$x = 41$

Solution: $T(n) = T\left(\frac{61}{82}n\right) + T\left(\frac{1}{41}n\right) + O(n)$ ■

h. **BYD**

$x = 101$

Solution: $T(n) = T\left(\frac{151}{202}n\right) + T\left(\frac{1}{101}n\right) + O(n)$ ■