Problem type 1:

Write the recurrence that describes the following problem. The recurrence is the piece-wise function that describes the calculation of the solution desired by the below problem. We *do not* want pseudocode.

(See variants below)

a. **BYA & BYH**

Given an array $A[1\,..\,n]$ of integers, compute the length of a *longest alternating subsequence*. A sequence $B[1\,..\,\ell]$ is *alternating* if $B[i] < B[i-1]$ for every even index $i \geq 2$, and $B[i] > B[i-1]$ for every odd index $i \geq 3$.

Let $LAS^+(i,j)$ denote the length of the longest alternating subsequence of $A[i\,..\,n]$ whose first element (if any) is larger than $A[j]$ and whose second element (if any) is smaller than its first. Let $LAS^-(i,j)$ denote the length of the longest alternating subsequence of $A[i\,..\,n]$ whose first element (if any) is smaller than $A[j]$ and whose second element (if any) is larger than its first.

Solution:

$$LAS^+(i,j) = \begin{cases} 0 & \text{if } i > n \\ LAS^+(i+1,j) & \text{if } i \leq n \text{ and } A[i] \leq A[j] \\ \max\left\{LAS^+(i+1,j), 1 + LAS^-(i+1,i)\right\} & \text{otherwise} \end{cases}$$

$$LAS^-(i,j) = \begin{cases} 0 & \text{if } i > n \\ LAS^-(i+1,j) & \text{if } i \leq n \text{ and } A[i] \geq A[j] \\ \max\left\{LAS^-(i+1,j), 1 + LAS^+(i+1,i)\right\} & \text{otherwise} \end{cases}$$

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b. **BYC & BYE**

Given an array $A[1\,..\,n]$ of integers, compute the length of a *longest decreasing subsequence*.

Let $LDS(i,j)$ denote the length of the longest decreasing subsequence of $A[i\,..\,n]$ where every element is smaller than $A[j]$.

Solution:

$$LDS(i,j) = \begin{cases} 0 & \text{if } i > n \\ LDS(i+1,j) & \text{if } i \leq n \text{ and } A[j] \leq A[i] \\ \max\left\{LDS(i+1,j), 1 + LDS(i+1,i)\right\} & \text{otherwise} \end{cases}$$

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c. **BYD & BYG**

Given an array $A[1..n]$, compute the length of a longest *palindrome* subsequence of A. Recall that a sequence $B[1..\ell]$ is a *palindrome* if $B[i] = B[\ell - i + 1]$ for every index i.

Let $LPS(i, j)$ denote the length of the longest palindrome subsequence of $A[i..j]$.

Solution:

$$LPS(i,j) = \begin{cases} 0 & \text{if } i > j \\ 1 & \text{if } i = j \\ \max\left\{ \begin{array}{c} LPS(i+1,j) \\ LPS(i,j-1) \end{array} \right\} & \text{if } i < j \text{ and } A[i] \neq A[j] \\ \max\left\{ \begin{array}{c} 2 + LPS(i+1,j-1) \\ LPS(i+1,j) \\ LPS(i,j-1) \end{array} \right\} & \text{otherwise} \end{cases}$$

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d. **BYB & BYF**

Given an array $A[1..n]$ of integers, compute the length of a longest *convex* subsequence of A. A sequence $B[1..\ell]$ is *convex* if $B[i] - B[i-1] > B[i-1] - B[i-2]$ for every index $i \geq 3$.

Let $LCS(i, j)$ denote the length of the longest convex subsequence of $A[i..n]$ whose first two elements are $A[i]$ and $A[j]$.

Solution:

$$LCS(i,j) = 1 + \max\left\{ LCS(j,k) \;\middle|\; j < k \leq n \text{ and } A[i] + A[k] > 2A[j] \right\}$$

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