Problem type 1:

Answer the following question:

> *(See variants below)*

a. **BYH**

How many strongly connected components can a direct acyclic graph have?

> **Solution:** A DAG must have n strongly connected components (each node is its own SCC). ■

b. **BYF**

How many topological sorts does a fully connected directed graph have?

> **Solution:** Trick question. Answer is 0. (one caveat, if $n = 1$, then the number of topological sorts is 1). ■

c. **BYE**

What type of graph has the greatest number of topological sorts?

> **Solution:** A completely disconnected graph (no edges), has $n!$ topological sorts. ■

d. **BYD**

Given a directed graph, give a algorithm that finds the node that has the largest reach (find u such that $|rch(u)|$ is maximized).

> **Solution:** Need to find vertex with larghest interval. Larger the interval means the more vertices were reached after this vertex.
>
> - Run DFS with pre/post numbering on G
> - return the vertex with the largest *post-pre* number
>
> Can't just look at vertex with largest post number because there may be multiple source vertices. ■

e. **BYB**

Given a directed graph, give a algorithm that finds the node that has the smallest reach (find u such that $|rch(u)|$ is minimized).

> **Solution:** Need to find a vertex in the sink SCC.
>
> - Run DFS with pre/post numbering on G^{rev}
> - return the vertex with the largest post number
>
> ■

f. **BYG**

You run DFS with pre/post numbering on a directed acyclic graph. You get the numbering for vertices u and v. You notice that the edge (u, v) can be classsified as a **forward** edge because of the relationship of the pre/post numberings.

Fill in the equality:

$$w < x < y < z$$

that must be true for a forward edge where

$$w, x, y, z, \in \{pre(u), post(u), pre(v), post(v)\}$$

Solution: $pre(u) < pre(v) < post(v) < post(u)$ ■

g. **BYC**

You run DFS with pre/post numbering on a directed acyclic graph. You get the numbering for vertices u and v. You notice that the edge (u, v) can be classsified as a **backward** edge because of the relationship of the pre/post numberings.

Fill in the equality:

$$w < x < y < z$$

that must be true for a forward edge where

$$w, x, y, z, \in \{pre(u), post(u), pre(v), post(v)\}$$

Solution: $pre(v) < pre(u) < post(u) < post(v)$ ■

h. **BYA**

You run DFS with pre/post numbering on a directed acyclic graph. You get the numbering for vertices u and v. You notice that the edge (u, v) can be classsified as a **cross** edge because of the relationship of the pre/post numberings.

Fill in the equality:

$$w < x < y < z$$

that must be true for a forward edge where

$$w, x, y, z, \in \{pre(u), post(u), pre(v), post(v)\}$$

Solution: $pre(u) < post(u) < pre(v) < post(v)$ or $pre(v) < post(v) < pre(u) < post(u)$ ■