Problem type 1:

You are given a known problem and unknown problem X. You can show the reduction listed below.

(See variants below)

Out of the following complexity classes:

P NP NP-hard NP-complete ExpTime

what classes **may** X belong to? In 1-2 sentences, explain why you made your particular selection(s). Assume $P \neq NP$.

a. **BYF**

LIS: Given a sequence A and an integer k, return TRUE if the longest increasing subsequence is more than k in length. FALSE otherwise.

Reduction: LIS $\leq_P$ X

Solution: We know LIS is in P. The reduction is saying that X is atleast in P, which isn't saying much and X may or may not belong to any complexity class.

(P) (NP) (NP-hard) (NP-complete) (ExpTime)

■

b. **BYA**

LIS: Given a sequence A and an integer k, return TRUE if the longest increasing subsequence is more than k in length. FALSE otherwise.

Reduction: X $\leq_P$ LIS

Solution: We know LIS is in P. The reduction is saying that X is atmost in P, which makes it belong to P/NP/EXPTIME, but not NP-hard since $P \neq NP$.

(P) (NP) NP-hard NP-complete (ExpTime)

■

c. **BYD**

SAT: Given a conjunctive normal formula, determine if there is a truth assignment that makes the formula evaluate to true.

Reduction: SAT $\leq_P$ X

Solution: We know SAT is NP-complete. The reduction is saying that X is atleast NP-hard, but the problem can be in NP, EXPTIME, etc. This makes it belong to the following complexity classes:

P NP NP-hard NP-complete ExpTime

■

d. **BYC**

SAT: Given a conjunctive normal formula, determine if there is a truth assignment that makes the formula evaluate to true.

Reduction: X $\leq_P$ SAT

Solution: We know SAT is NP-complete. The reduction is saying that X is no harder than NP-hard, which means that the problem must be in NP. This means it may belong to any of the complexity classes.

P NP NP-hard NP-complete ExpTime

■

Problem type 2:

You are given a known problem and unknown problem X. You can show the reduction listed below.

(See variants below)

Out of the following complexity classes:

P NP NP-hard NP-complete ExpTime

what classes **must** X belong to? In 1-2 sentences, explain why you made your particular selection(s). Assume $P \neq NP$.

a. **BYE**

LIS: Given a sequence A and an integer k, return TRUE if the longest increasing subsequence is more than k in length. FALSE otherwise.

Reduction: LIS $\leq_P$ X

Solution: We know LIS is in P. The reduction is saying that X is atleast in P, which isn't saying much and X may or may not belong to any complexity class.

P NP NP-hard NP-complete ExpTime

■

b. **BYG**

LIS: Given a sequence A and an integer k, return TRUE if the longest increasing subsequence is more than k in length. FALSE otherwise.

Reduction: X $\leq_P$ LIS

Solution: We know LIS is in P. The reduction is saying that X is atmost in P, which makes it belong to everything else as well.

(P) (NP) NP-hard NP-complete (ExpTime)

■

c. **BYH**

SAT: Given a conjunctive normal formula, determine if there is a truth assignment that makes the formula evaluate to true.

Reduction: SAT $\leq_P$ X

Solution: We know SAT is NP-complete. The reduction is saying that X is atleast NP-hard, the issue is that X may not even be decidable! That means all we know is that it belongs to the following complexity classes:

P NP (NP-hard) NP-complete ExpTime

■

d. **BYB**

SAT: Given a conjunctive normal formula, determine if there is a truth assignment that makes the formula evaluate to true.

Reduction: X $\leq_P$ SAT

Solution: We know SAT is NP-complete. The reduction is saying that X is no harder than NP-hard, which means that the problem must be in NP. This means it may belong to any of the complexity classes.

P (NP) NP-hard NP-complete (ExpTime)

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