Problem type 1:

Briefly describe a reduction that shows the following:

(See variants below)

Assume $P \neq NP$

a. **BYD/BYG**

3Color: (**3Color(G)**)

- Input: A undirected graph G
- Output: True if the vertices in G can be marked with **3** colors such that no adjacent vertices share the same color, False otherwise

4Color: (**4Color(G')**)

- Input: A undirected graph G'
- Output: True if the vertices in G' can be marked with **4** colors such that no adjacent vertices share the same color, False otherwise

Reduction: 3Color $\leq_P$ 4Color

Solution: Add a vertex to G and connect it too all vertices. This will form G'. Doing this consumes one of the four colors the 4Color algorithm may use to color and thus return true if and only if G is 3-colorable. ■

b. **BYA/BYH**

SAT: (**SAT(ϕ)**)

- Input: A conjunctive normal formula ϕ
- Output: True if there exists a truth assignment that let's ϕ evaluate to True, False otherwise

AlmostSAT: (**AlmostSAT(ϕ')**)

- Input: A conjunctive normal formula ϕ'
- Output: True if there exists a truth assignment that satisfies **all but one** clauses in ϕ', False otherwise

Reduction: SAT $\leq_P$ AlmostSAT

Solution: Make $\phi' = \phi \wedge (x) \wedge (\neg x)$. Only one of the two added clauses can ever be satisfied which means all but one clauses in ϕ' can be satisifed only if ϕ can be satisfied. ■

c. **BYC/BYE**

Hamiltonian Path: (**HamPath(G)**)

- INPUT: A undirected graph G
- OUTPUT: True if there exists a simple path that visits all vertices exactly once, False otherwise

Almost Hamiltonian Path: (**AlmostHamPath(G')**)

- INPUT: A undirected graph G
- OUTPUT: True if there exists a simple path that visits **all but one** vertices exactly once, False otherwise

Reduction: HamPath $\leq_P$ AlmostHamPath

> **Solution:** Make $G' = \langle V \cup x, E \rangle$. This new vertex isn't connected to anythign whihc means that the only way to get a path at covers all but one vertices in G' is to have a path that visits every vertex in G. ∎

d. **BYB/BYF**

Undirected Hamiltonian Path: (**UndirHamPath(G)**)

- INPUT: A undirected graph G
- OUTPUT: True if there exists a simple path that visits all vertices exactly once, False otherwise

Directed Hamiltonian Path: (**DirHamPath(G')**)

- INPUT: A directed graph G'
- OUTPUT: True if there exists a simple path that visits all vertices exactly once, False otherwise

Reduction: UndirHamPath $\leq_P$ DirHamPath

> **Solution:** $G' = \langle V, (u,v) + (v,u) \rangle \forall \{u,v\} \in G$ ∎