CS/ECE-374-B: Algorithms and Models of Computation, Spring 2024

Midterm exam 1 – February 15, 2024

- You can do hard things! Grades do matter, but not as much as you may think, but then life is uncertain anyway, so what.
- **Don't cheat.** The consequence for cheating is far greater than the reward. Just try your best and you'll be fine.
- **Please read the entire exam before writing anything.** There are 8 problems and most have multiple parts.
- This is a closed-book exam. At the end of the exam, you'll find a multi-page cheat sheet. *Do not tear out the cheat sheet!* No outside material is allowed on this exam.
- You should write your answers legibly and in the space given for the question. Overly verbose answers will be penalized.
- Scratch paper is available on the back of the exam. *Do not tear out the scratch paper!* It messes with the auto-scanner.
- **You have 75 minutes (1.25 hours) for the exam.** Manage your time well. *Do not spend too much time on questions you do not understand and focus on answering as much as you can!*

Name: ______________________________

NetID: ______________________________

Date: ______________________________

Problem 1 [20 points]

a. Write the recursive definition for the following language. [6]

$$L_a = \left\{ w \mid w \in \{0,1\}^*,\ |w| = 2n \text{ for some } n \geq 0, \text{ and } w = w^R \text{ where } w^R \text{ is the reverse of } w. \right\}$$

Solution: L_a represents the language of all even-length binary palindromes. The recursive definition is given as follows.

- $w = \epsilon$, or
- $w = axa$ for some $a \in \{0,1\}$ and $x \in L_a$.

■

b. Write regular expressions for the following languages. [7+7]

i. $L_{bi} = \{w \mid w \in \{0,1\}^*,\ w$ does not contain the subsequence **00**.$\}$

Solution: L_{bi} represents the language of all binary strings with number of zeros less than or equal to 1. The regular expression is given as follows.

$$\epsilon + 1^* + 1^*01^*$$

■

ii. $L_{bii} = \{w \mid w \in \{0,1\}^*,\ w$ contains **00** and **11** as subsequences.$\}$

Solution: Let S be the set of all strings of length 4 that have the desired property. That is, we have the following.

$$S = \left\{ \mathtt{0011}, \mathtt{0101}, \mathtt{0110}, \mathtt{1001}, \mathtt{1100}, \mathtt{1010} \right\}.$$

There are $\binom{4}{2} = 6$, such strings, and they all belong to our language. For a string s, let $f(s)$ be the regular expression of inserting $(\mathtt{0} + \mathtt{1})^*$ between any two characters of s, and also in the beginning of s and the end of s. For example, we have $f(\mathtt{01}) = (\mathtt{0}+\mathtt{1})^*\mathtt{0}(\mathtt{0}+\mathtt{1})^*\mathtt{1}(\mathtt{0}+\mathtt{1})^*$. The desired expression is

$$\sum_{s \in S} f(s).$$

■

Problem 2 [10 points]

Consider the state diagram given in Figure 1 .

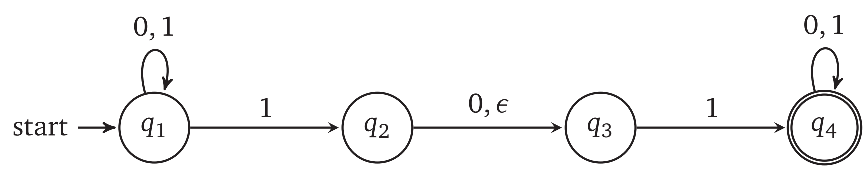


Figure 1.

a. Write the corresponding nondeterministic finite automaton in a formal manner. [5]

Solution: For the given state diagram, the corresponding finite automaton in a formal manner is as follows.

- $Q = \{q_1, q_2, q_3, q_4\}$.
- $\Sigma = \{0, 1\}$.
- δ is given as follows.

δ	0	1	ϵ
q_1	$\{q_1\}$	$\{q_1, q_2\}$	ϕ
q_2	$\{q_3\}$	ϕ	$\{q_3\}$
q_3	ϕ	$\{q_4\}$	ϕ
q_4	$\{q_4\}$	$\{q_4\}$	ϕ

- $q_0 = q_1$.
- $F = \{q_4\}$.

■

b. What ALL sequences of states does the above machine go through on inputs **010** and **010110**? Note that there may be multiple sequences of states for the same input. Does the machine accept these inputs? **Hint.** Draw the computation tree for each input. [5]

Solution: The required sequences of states for **010** and **010110** are given in the computation trees in Figures 2 and 3, respectively. Furthermore, based on those trees, we conclude that the machine does not accept **010** and accepts **010110**. ■

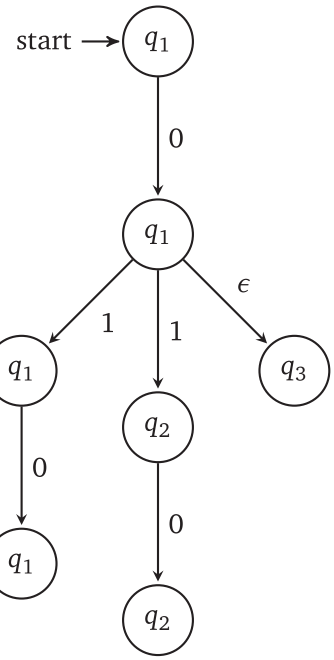


Figure 2. Computation tree for `010`.

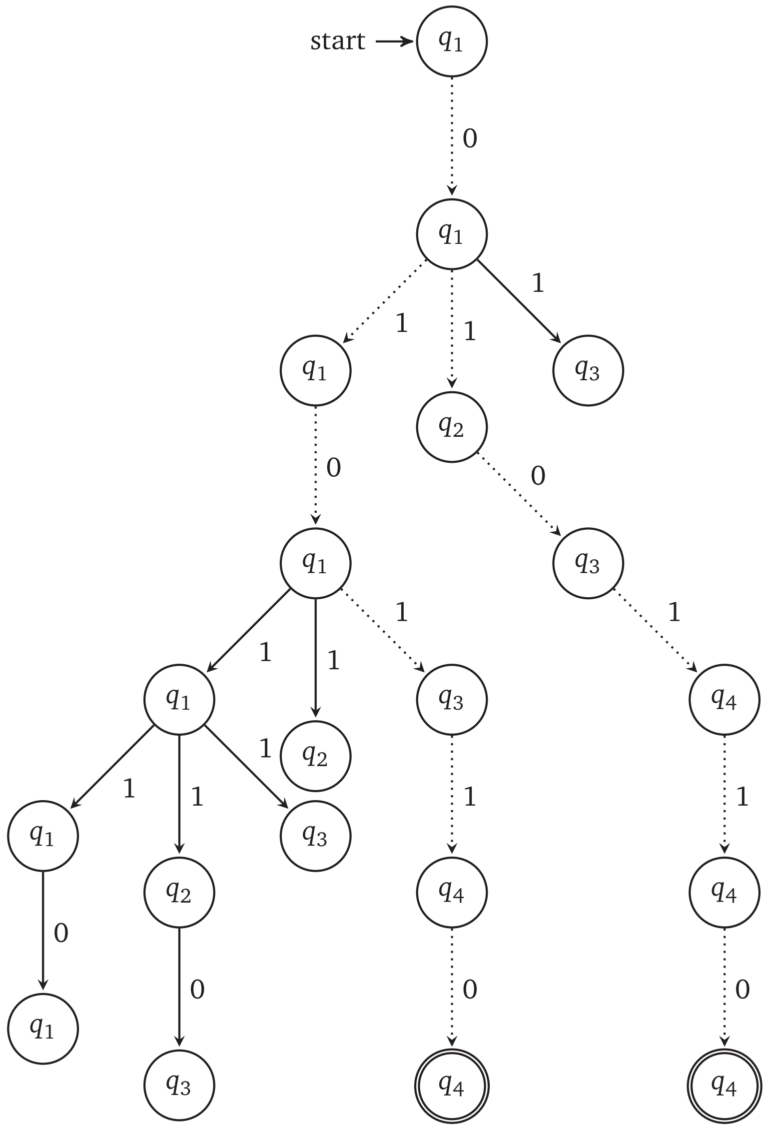


Figure 3. Computation tree for `010110`.

Problem 3 [20 points]

a. Convert the NFA given in Figure 4 to an equivalent DFA. [10]

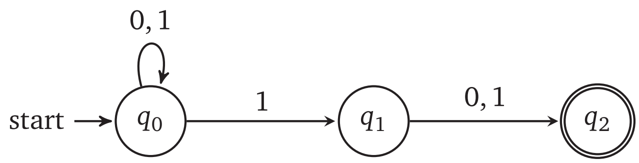


Figure 4.

Solution: The formal representation of the above NFA upon conversion (using the incremental method) to a DFA is as follows.

(a) $Q = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_2\}\}$.

(b) $\Sigma = \{0, 1\}$.

(c) $\delta : Q \times \Sigma \to Q$ is the transition function given as follows.

δ	0	1
$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_2\}$	$\{q_0\}$	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

(d) $q_0 = \{q_0\}$.

(e) $F = \{\{q_0, q_2\}, \{q_0, q_1, q_2\}\}$.

■

b. For $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, convert the regular expression $(\mathtt{a} + \mathtt{b})^*\mathtt{aba}$ to an equivalent NFA. [10]

Solution: The step-by-step construction of an equivalent NFA is shown in Figure 5. Interpret $\cup$ as $+$. ■

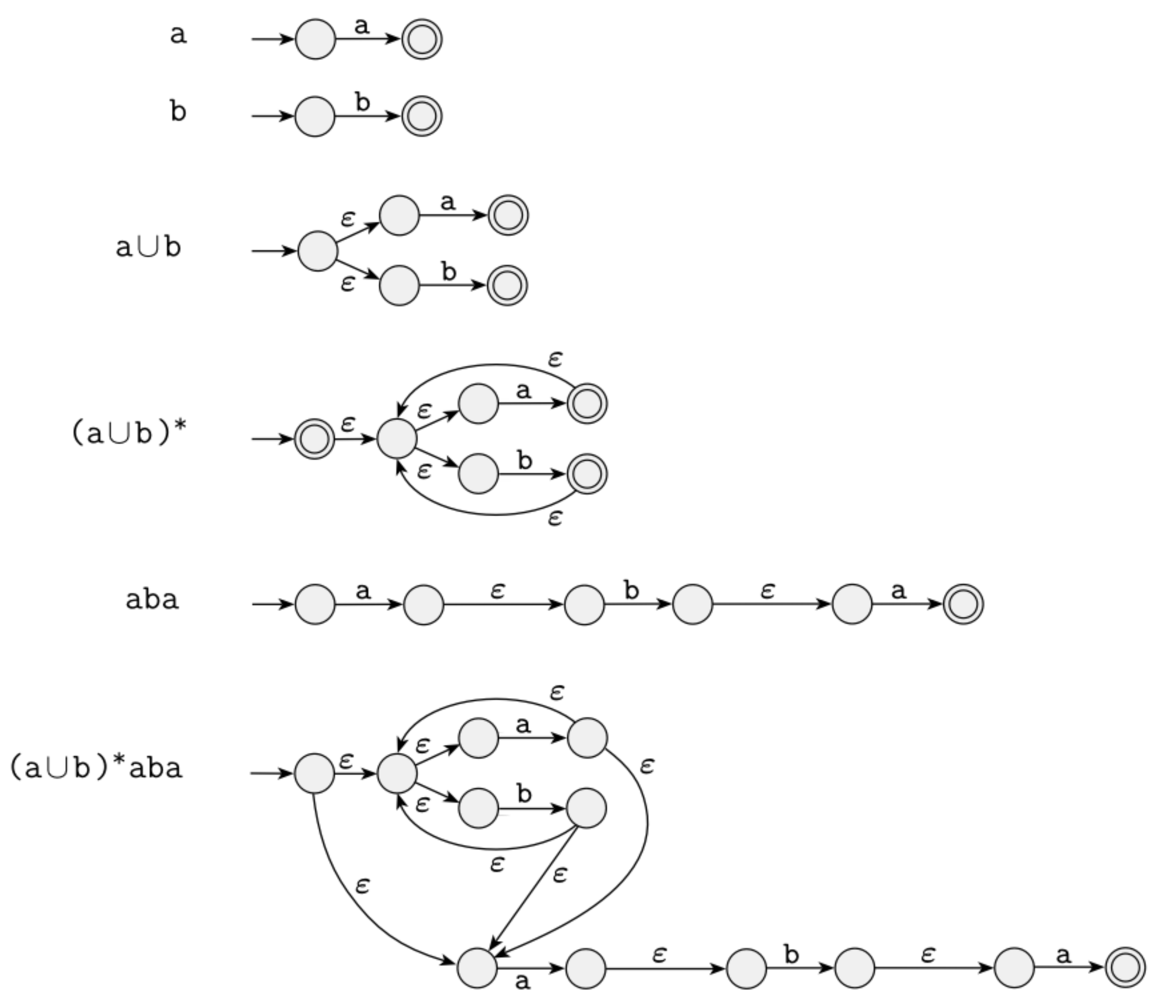


Figure 5. Regular expression to NFA.

Problem 4 [10 points]

Given a regular language L over $\{0,1\}^*$, prove that the language $L' := \{xy \mid x\mathbf{1}y \in L\}$ is regular. [10]

Solution: Intuitively, L' is the set of all strings that can be obtained from strings in L by deleting exactly one **1**. For example, if $L = \{\mathbf{101101}, \mathbf{00}, \varepsilon\}$, then $L' = \{\mathbf{01101}, \mathbf{10101}, \mathbf{10110}\}$.

Let $M = (Q, s, A, \delta)$ be a DFA that accepts L. We construct an NFA $M' = (Q', s', A', \delta')$ with ε-transitions that accepts L' as follows.

Intuitively, M' simulates M, but inserts a single **1** into M's input string at a nondeterministically chosen location.

- The state (q, before) means (the simulation of) M is in state q and M' has not yet inserted a **1**.
- The state (q, after) means (the simulation of) M is in state q and M' has already inserted a **1**.

$$Q' := Q \times \{\mathit{before}, \mathit{after}\}$$
$$s' := (s, \mathit{before})$$
$$A' := \big\{(q, \mathit{after}) \bigm| q \in A\big\}$$

$$\delta'((q, \mathit{before}), \varepsilon) = \big\{(\delta(q, \mathbf{1}), \mathit{after})\big\}$$
$$\delta'((q, \mathit{after}), \varepsilon) = \varnothing$$

$$\delta'((q, \mathit{before}), a) = \big\{(\delta(q, a), \mathit{before})\big\}$$
$$\delta'((q, \mathit{after}), a) = \big\{(\delta(q, a), \mathit{after})\big\}$$

■

Problem 5 [15 points]

Consider the PDA, P given in Figure 6.

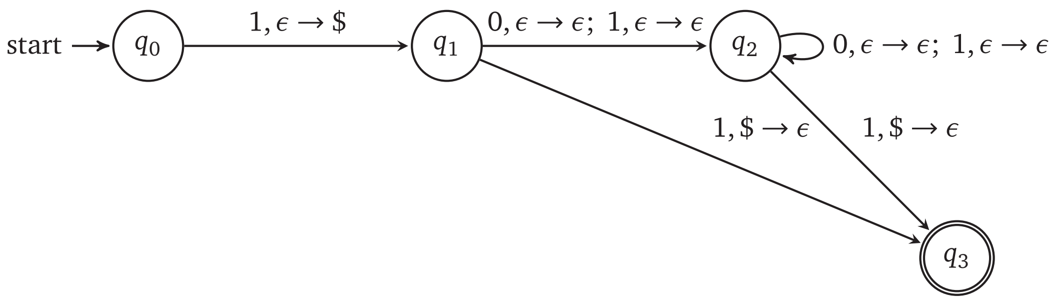


Figure 6.

a. Does the above PDA accept the following strings? [4]

i. **1010**
ii. **0101**
iii. **1001**
iv. **1011**

Solution: The solution is given as follows.

i. No.
ii. No.
iii. Yes.
iv. Yes.

■

b. Describe $L(P)$ in one sentence. [3]

Solution: All binary strings of length greater than 1 that start with **1** and end with **1**. ■

c. Give a context free grammar that generates $L(P)$. [4]

Solution: The context free $G = (V, T, P, S)$ that generates $L(P)$ is given as follows.

- $V = \{\texttt{S}, \texttt{A}\}$
- $T = \{0, 1\}$
- $P : \texttt{S} \rightarrow \texttt{1A1},\ \texttt{A} \rightarrow \epsilon \mid \texttt{0A} \mid \texttt{1A}$
- $S = \texttt{S}$

■

d. Which one of the following statements is true? [4]

i. $L(P)$ is context sensitive but not regular.
ii. $L(P)$ is not context sensitive but regular.
iii. $L(P)$ is both context sensitive and regular.
iv. $L(P)$ is neither context sensitive nor regular.

Prove the statement you chose.

Solution: The true statement is iii, i.e., $L(P)$ is both context sensitive and regular. The regular expression $1(0+1)^*1$ generates $L(P)$. Hence, $L(P)$ is regular, and hence, context-sensitive. ■

Problem 6 [10 points]

Consider a context free grammar $G = (V, T, P, S)$, where

- $V = \{\mathsf{B}, \mathsf{E}\}$
- $T = \{\mathsf{a}, \mathsf{b}\}$
- $P = \{\mathsf{B} \rightarrow \mathsf{a} \mid \mathsf{aEb};\ \mathsf{E} \rightarrow \epsilon \mid \mathsf{Ea}\}$
- $S = \mathsf{B}$

Construct a PDA that recognizes $L(G)$. [10]

Solution: The PDA that recognizes $L(G)$ is given in Figure 7. ■

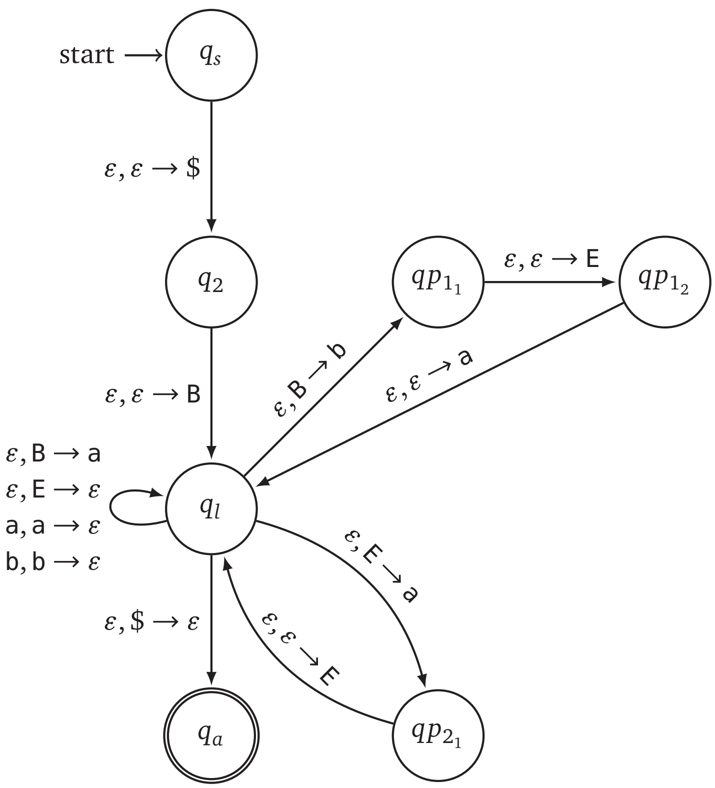


Figure 7. CFG to PDA.

Problem 7 [5 points]

Let $L_1, \ldots, L_n$ be some regular languages and L_k be a non-regular language such that

$$L_k = L_1 \oplus L_2 \oplus \cdots \oplus L_n \oplus L_u, \ \oplus \in \{\cup, \cap, \cdot\},$$

for some L_u then (formally) prove that L_u is non-regular.

Solution: By way of contradiction, assume that L_u is regular. Hence, $L_k = L_1 \oplus L_2 \oplus \cdots \oplus L_n \oplus L_u, \ \oplus \in \{\cup, \cap, \cdot\}$ is regular as regular languages are closed under union, intersection, and concatenation. But, L_k is given to be non-regular. Hence, a contradiction. This implies that L_u is regular. ■

Problem 8 [10 points]

For each of the following languages defined over $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$, prove if it is regular or not. Furthermore, prove if it is context free or not.

a. $L_1 = \{\mathtt{a}^n\mathtt{b}^n \mid 0 \leq n \leq 3\}$ [2]

Solution: The solution is given as follows.

- L_1 is a finite language. As every finite language is regular, L_1 is **regular**.
- As every regular language is also context free, L_1 is **context free**.

■

b. $L_2 = \{\mathtt{a}^n\mathtt{b}^m\mathtt{c}^n \mid m, n \geq 0\}$ [4]

Solution: The solution is given as follows.

- Consider the fooling set: $\{\mathtt{a}^i : i \geq 0\}$. Note that, for $i \neq j$, $m \geq 0$, $\mathtt{a}^i\mathtt{b}^m\mathtt{c}^i \in L_2$ but $\mathtt{a}^j\mathtt{b}^m\mathtt{c}^i \notin L_2$. Hence, L_2 is **non-regular**.
- Consider the following context free grammar $G = (V, T, P, S)$, where
  - $V = \{\mathtt{S}, \mathtt{X}\}$
  - $T = \{\mathtt{a}, \mathtt{b}\}$
  - $P = \{\mathtt{S} \rightarrow \mathtt{aSc} \mid \mathtt{X} \mid \epsilon; \mathtt{X} \rightarrow \mathtt{bX} \mid \epsilon\}$
  - $S = \mathtt{S}$

  G generates L_2. Hence, L_2 is **context free**.

■

c. $L_3 = \{\mathtt{a}^n\mathtt{b}^m \mid n > m \text{ or } m > n\}$ [4]

Solution: The solution is given as follows.

- Consider the fooling set: $\{\mathtt{a}^i : i \geq 0\}$. Note that, for $i \neq j$, $\mathtt{a}^i\mathtt{b}^i \notin L_3$ but $\mathtt{a}^j\mathtt{b}^i \in L_3$. Hence, L_3 is **non-regular**.
- Consider the following context free grammar $G = (V, T, P, S)$, where
  - $V = \{\mathtt{S}, \mathtt{X}, \mathtt{Y}\}$
  - $T = \{\mathtt{a}, \mathtt{b}\}$
  - $P = \{\mathtt{S} \rightarrow \mathtt{aSb} \mid \mathtt{X} \mid \mathtt{Y}; \mathtt{X} \rightarrow \mathtt{aX} \mid \mathtt{a}; \mathtt{Y} \rightarrow \mathtt{bY} \mid \mathtt{b}\}$
  - $S = \mathtt{S}$

  G generates L_3. Hence, L_3 is **context free**.

■

This page is for additional scratch work!

ECE 374 B Language Theory: Cheatsheet

1 Languages and strings

Languages

Definitions

- An *alphabet* Σ is a **finite** set of symbols.
- A *string* in Σ^* is a **finite** sequence of symbols in Σ.
- A *language* is L is a set of strings over some alphabet.

All languages represent mathematical problems.
Example: multiplication of two integers:

$$L_{MULT2} = \begin{Bmatrix} 1 \times 1|1, & 1 \times 2|2, & 1 \times 3|3, \ldots \\ 2 \times 1|2, & 2 \times 2|4, & 2 \times 3|6, \ldots \\ \vdots & \vdots & \vdots \\ n \times 1|n, & n \times 2|2n, & n \times 3|3n, \ldots \end{Bmatrix} \quad (1)$$

Language operations

- For languages A, B the *concatenation* of A, B is $AB = \{xy \mid x \in A, y \in B\}$.
- For languages A, B, their *union* is $A \cup B$, *intersection* is $A \cap B$, and *difference* is $A \setminus B$ (also written as $A - B$).
- For language $A \subseteq \Sigma^*$ the *complement* of A is $\bar{A} = \Sigma^* \setminus A$.
- Σ^n is the set of all strings of length n.
- $\Sigma^* = \cup_{n \geq 0} \Sigma^n$ is the set of all strings over Σ.
- $\Sigma^+ = \cup_{n \geq 1} \Sigma^n$ is the set of non-empty strings over Σ.

Strings

Definitions

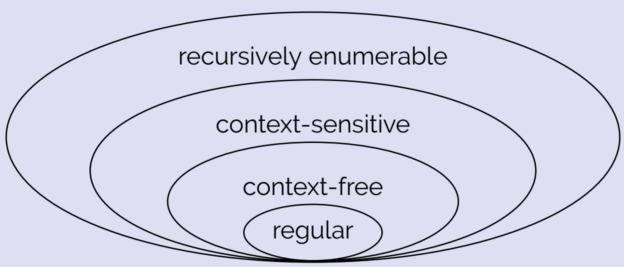
- The *length* of a string w (denoted by $|w|$) is the number of symbols in w.
- For integer $n \geq 0$, Σ^n is set of all strings over Σ of length n. Σ^* is the set of all strings over Σ.
- Σ^* is the set of all strings of all lengths including empty string.
- ε is a *string* containing no symbols.
- $\varnothing$ is the *empty set*. It contains no strings.

String operations

- If x and y are strings then xy denotes their concatenation. Recursively:
  - $xy = y$ if $x = \varepsilon$
  - $xy = a(wy)$ if $x = aw$
- v is *substring* of $w \iff$ there exist strings x, y such that $w = xvy$.
  - If $x = \varepsilon$ then v is a *prefix* of w
  - If $y = \varepsilon$ then v is a *suffix* of w
- A *subsequence* of a string $w = w_1 w_2 \ldots w_n$ is either a subsequence of $w_2 \ldots w_n$ or w_1 followed by a subsequence of $w_2 \ldots w_n$.
- If w is a string then w^n is defined inductively as follows: $w^n = \varepsilon$ if $n = 0$ or $w^n = ww^{n-1}$ if $n > 0$

2 Overview of language complexity

Overview



Grammar	Languages	Production Rules	Automaton	Examples
Type-0	recursively enumerable	$\gamma \to \alpha$ (no constraints)	Turing machine	$L = \{w \mid w$ is a *TM* which halts$\}$
Type-1	context-sensitive	$\alpha A \beta \to \alpha \gamma \beta$	linear bounded nondeterministic Turing machine	$L = \{a^n b^n c^n \mid n > 0\}$
Type-2	context-free	$A \to \alpha$	nondeterministic pushdown automata	$L = \{a^n b^n \mid n > 0\}$
Type-3	regular	$A \to aB$	finite state machine	$L = \{a^n \mid n > 0\}$

Meaning of symbols:
- a - terminal
- A, B - variables
- α, β, γ - strings in $\{a \cup A\}^*$ where α, β are maybe empty, γ is never empty

[a]

[a] Table borrowed from Wikipedia: `https://en.wikipedia.org/wiki/Chomsky_hierarchy`

3 Regular languages

Regular language - overview

A language is regular if and only if it can be obtained from finite languages by applying

- union,
- concatenation or
- Kleene star

finitely many times. All regular languages are representable by regular grammars, DFAs, NFAs and regular expressions.

Regular expressions

Useful shorthand to denotes a language.
A *regular expression* $\mathbf{r}$ over an alphabet Σ is one of the following:
Base cases:

- $\varnothing$ the language $\varnothing$
- ε denotes the language $\{\varepsilon\}$
- a denote the language $\{a\}$

Inductive cases: If $\mathbf{r_1}$ and $\mathbf{r_2}$ are regular expressions denoting languages L_1 and L_2 respectively (i.e.,$L(\mathbf{r_1}) = L_1$ and $L(\mathbf{r_2}) = L_2$) then,

- $\mathbf{r_1} + \mathbf{r_2}$ denotes the language $L_1 \cup L_2$
- $\mathbf{r_1} \cdot \mathbf{r_2}$ denotes the language $L_1 L_2$
- $\mathbf{r_1^*}$ denotes the language L_1^*

Examples:

- 0^* - the set of all strings of 0s, including the empty string
- $(00000)^*$ - set of all strings of 0s with length a multiple of 5
- $(0+1)^*$ - set of all binary strings

Nondeterministic finite automata

NFAs are similar to DFAs, but may have more than one transition destination for a given state/character pair.

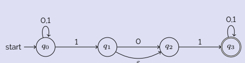
An NFA N *accepts a string* w iff some accepting state is reached by N from the start state on input w.

The *language accepted (or recognized)* by an NFA N is denoted $L(N)$ and defined as $L(N) = \{w \mid N \text{ accepts } w\}$.

A *nondeterministic finite automaton (NFA)* $N = (Q, \Sigma, s, A, \delta)$ is a five tuple where

- Q is a finite set whose elements are called *states*
- Σ is a finite set called the *input alphabet*
- $\delta : Q \times \Sigma \cup \{\varepsilon\} \to \mathcal{P}(Q)$ is the *transition function* (here $\mathcal{P}(Q)$ is the power set of Q)
- s and Σ are the same as in DFAs

Example:



- $Q = \{q_0, q_1, q_2, q_3\}$
- $\Sigma = \{0, 1\}$
- δ :

	ε	0	1
q_0	$\{q_0\}$	$\{q_0\}$	$\{q_0, q_1\}$
q_1	$\{q_1, q_2\}$	$\{q_2\}$	$\varnothing$
q_2	$\{q_2\}$	$\varnothing$	$\{q_3\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3\}$

- $s = q_0$
- $A = \{q_3\}$

For NFA $N = (Q, \Sigma, \delta, s, A)$ and $q \in Q$, the ε-reach(q) is the set of all states that q can reach using only ε-transitions.
Inductive definition of $\delta^* : Q \times \Sigma^* \to \mathcal{P}(Q)$:

- if $w = \varepsilon$, $\delta^*(q, w) = \varepsilon\text{-reach}(q)$
- if $w = a$ for $a \in \Sigma$, $\delta^*(q, a) = \varepsilon\text{reach}\left(\bigcup_{p \in \varepsilon\text{-reach}(q)} \delta(p, a)\right)$
- if $w = ax$ for $a \in \Sigma, x \in \Sigma^*$: $\delta^*(q, w) = \varepsilon\text{reach}\left(\bigcup_{p \in \varepsilon\text{-reach}(q)} \left(\bigcup_{r \in \delta^*(p,a)} \delta^*(r, x)\right)\right)$

Regular closure

Regular languages are closed under union, intersection, complement, difference, reversal, Kleene star, concatenation, etc.

Deterministic finite automata

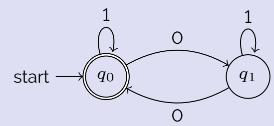
DFAs are finite state machines that can be represented as a directed graph or in terms of a tuple.

The *language accepted* (or recognized) by a DFA M is denoted by $L(M)$ and defined as $L(M) = \{w \mid M \text{ accepts } w\}$.

A *deterministic finite automaton (DFA)* $M = (Q, \Sigma, s, A, \delta)$ is a five tuple where

- Q is a finite set whose elements are called *states*
- Σ is a finite set called the *input alphabet*
- $\delta : Q \times \Sigma \to Q$ is the *transition function*
- $s \in Q$ is the *start state*
- $A \subseteq Q$ is the set of *accepting/final* states

Example:



- $Q = \{q_0, q_1\}$
- $\Sigma = \{0, 1\}$
- δ :

	0	1
q_0	q_1	q_0
q_1	q_0	q_1

- $s = q_0$
- $A = \{q_0\}$

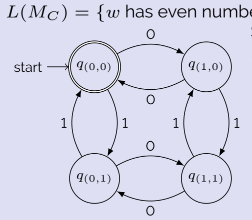
Every string has a unique walk along a DFA. We define the extended transition function as $\delta^* : Q \times \Sigma^* \to Q$ defined inductively as follows:

- $\delta^*(q, w) = q$ if $w = \varepsilon$
- $\delta^*(q, w) = \delta^*(\delta(q, a), x)$ if $w = ax$.

Can create a larger DFA from multiple smaller DFAs. Suppose

- $L(M_0) = \{w \text{ has an even number of 0s}\}$ (pictured above) and
- $L(M_1) = \{w \text{ has an even number of 1s}\}$.

$L(M_C) = \{w \text{ has even number of 0s and 1s}\}$

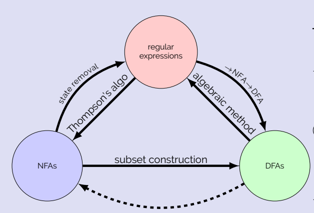


Suppose $M_0 = (Q_0, \Sigma, s_0, A_0, \delta_0)$ and $M_1 = (Q_1, \Sigma, s_1, A_1, \delta_1)$. Then

- $Q = Q_0 \times Q_1 = \{(q_0, q_1) \mid q_0 \in Q_0, q_1 \in Q_1\}$
- $s = (s_0, s_1)$
- $\delta : Q \times \Sigma \to Q$, where $\delta((q_0, q_1), a) = (\delta_0(q_0, a), \delta_1(q_1, a))$
- $A = \{(q_0, q_1) \mid q_0 \in A_0 \text{ and } q_1 \in A_1\}$

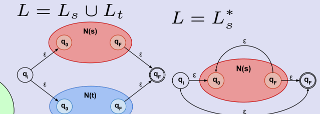
Regular language equivalences

A regular language can be represented by a regular expression, regular grammar, DFA and NFA.

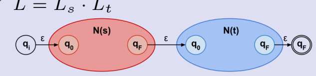


Thompson's algorithm:

$L = L_s \cup L_t$ $\quad$ $L = L_s^*$



$L = L_s \cdot L_t$



Arden's rule: If $R = Q + RP$ then $R = QP^*$.

Fooling sets

Some languages are not regular (Ex. $L = \{0^n 1^n \mid n \geq 0\}$).

Two states $p, q \in Q$ are *distinguishable* if there exists a string $w \in \Sigma^*$, such that

$\delta^*(p, w) \in A$ and $\delta^*(q, w) \notin A$.

or

$\delta^*(p, w) \notin A$ and $\delta^*(q, w) \in A$.

Two states $p, q \in Q$ are *equivalent* if for all strings $w \in \Sigma^*$, we have that

$$\delta^*(p, w) \in A \iff \delta^*(q, w) \in A.$$

For a language L over Σ a set of strings F (could be infinite) is a *fooling set* or *distinguishing set* for L if every two distinct strings $x, y \in F$ are distinguishable.

4 Context-free languages

Context-free languages

A language is context-free if it can be generated by a context-free grammar. A context-free grammar is a quadruple $G = (V, T, P, S)$

- V is a finite set of *nonterminal (variable) symbols*
- T is a finite set of *terminal symbols* (alphabet)
- P is a finite set of *productions*, each of the form $A \to \alpha$ where $A \in V$ and α is a string in $(V \cup T)^*$ Formally, $P \subseteq V \times (V \cup T)^*$.
- $S \in V$ is the *start symbol*

Example: $L = \{ww^R | w \in \{0,1\}^*\}$ is described by $G = (V, T, P, S)$ where V, T, P and S are defined as follows:

- $V = \{S\}$
- $T = \{0, 1\}$
- $P = \{S \to \varepsilon \mid 0S0 \mid 1S1\}$
  (abbreviation for $S \to \varepsilon$, $S \to 0S0$, $S \to 1S1$)
- $S = S$

Pushdown automata

A pushdown automaton is an NFA with a stack.

The language $L = \{0^n 1^n \mid n \geq 0\}$ is recognized by the pushdown automaton:

A *nondeterministic pushdown automaton (PDA)* $P = (Q, \Sigma, \Gamma, \delta, s, A)$ is a **six** tuple where

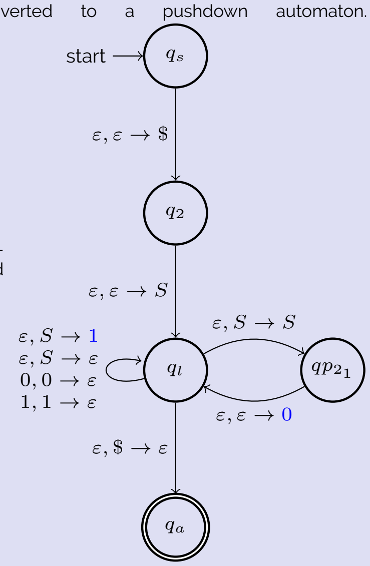
- Q is a finite set whose elements are called *states*
- Σ is a finite set called the *input alphabet*
- Γ is a finite set called the *stack alphabet*
- $\delta : Q \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to \mathcal{P}(Q \times (\Gamma \cup \{\varepsilon\}))$ is the *transition function*
- s is the start state
- A is the set of accepting states

In the graphical representation of a PDA, transitions are typically written as $\langle$input read$\rangle$, $\langle$stack pop$\rangle$ $\to$ $\langle$stack push$\rangle$.

A CFG can be converted to a pushdown automaton.

The PDA to the right recognizes the language described by the following grammar:

$$S \to 0S|1|\varepsilon$$



Context-free closure

Context-free languages are closed under union, concatenation, and Kleene star.

They are **not** closed under intersection or complement.