ECE 374 B ✧ Spring 2024
Homework 2

- **Submit your solutions electronically on the course Gradescope site as PDF files.** If you plan to typeset your solutions, please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.

- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.

  – Always give complete solutions, not just examples.
  – Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  – Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. This is a two part question. For the first part you'll be asked to come up with several (semi-)simple DFAs and for the next you'll be asked to formally combine those DFAs into one solution. Read the whole question before starting.

   (a) Describe the DFA that describes the following languages ($\Sigma = \{0, 1\}$). Formally define the DFAs and make sure their definitions are unique w.r.t. the other languages in 1.a (will make sense when doing part b):

      i. L_1 contains all strings where the substring 01 appears an odd number of times.

      ii. L_2 contains all strings where $\#(1, w)$ is divisible by three.

      iii. L_3 contains all strings where the binary value of w is divisible by seven.

   (b) Let L denote the set of all strings $w \in \{0, 1\}^*$ that are in **at most two** of the languages in part (a). Formally describe a DFA with input alphabet $\Sigma = \{0, 1\}$, that accepts the language L, by explicitly describing the states Q, the start state s, the accepting states A, and the transition function δ. Do not attempt to draw this DFA. At minimum, the smallest DFA for this language has 84 states.

      Argue your machine is correct by *concisely* explaining explaining your DFA (it's formal definition).

2. Let

$$\Sigma = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

   Consider each row to be a binary number and let

$$C = \{w \in \Sigma^* \mid \text{the bottom row of } w \text{ is three times the top row.}\}$$

   For example

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in D, \text{ but } \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \notin D.$$

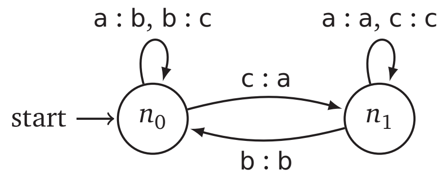
   Show that C is regular. (Hint, it is easier to to look at the matrices in reverse order).

3. Let B and C be languages over $\Sigma = \{0, 1\}$. Let $\xrightarrow{0}$ be an operation defined as the following:

$$B \xrightarrow{0} C = \{w \in C \mid \exists x \in B \text{ such that } \#(0, w) = \#(0, x)\}$$

   Show that the class of regular languages is closed under the $\xrightarrow{0}$ operation.

4. **Other types of automata:** A *finite-state transducer* (FST) is a type of deterministic finite automaton whose output is a string instead of just *accept* or *reject*. The following is the state diagram of finite state transducer FST_0.



Each transition of an FST is labeled at least an input symbol and an output symbol, separated by a colon (:). There can also be multiple input-output pairs for each transitions, separated by a comma (,). For instance, the transition from n_0 to itself can either take a or b as an input, and outputs b or c respectively.

When an FST computes on an input string $s := \overline{s_0 s_1 \ldots s_{n-1}}$ of length n, it takes the input symbols $s_0, s_1, \ldots, s_{n-1}$ one by one, starting from the starting state, and produces corresponding output symbols. For instance, the input string abccba produces the output string bcacbb, while cbaabc produces abbbca.

(a) Describe a formal model of FST. Specifically, describe the 5-tuple that defines an FST. *Hint: An* FST *has no accepting states, but it has the output alphabet* Γ. *Its transition function is of the form* $\delta : Q \times \Sigma \to Q \times \Gamma$.

(b) Give a formal description of FST_0.

(c) Give a state diagram of an FST with the following behavior. Its input and output alphabets are $\{\mathsf{T}, \mathsf{F}\}$. Its output string is inverted on the positions with indices divisible by 3 and is identical on all the other positions. For instance, on an input TFTTFTFT it should output FFTFFTTT.

5. **Another language transformation:** Let Σ be an arbitrary set, and L be a language defined over Σ. Define an operation cycle as the following:

$$\mathsf{cycle}(L) := \{xy | x, y \in \Sigma^*, yx \in L\}$$

Prove that the class of regular languages is closed under the operation cycle.