ECE 374 B ✧ Spring 2024

Homework 3

- **Submit your solutions electronically on the course Gradescope site as PDF files.** please use the LaTeX solution template on the course web site. If you must submit scanned handwritten solutions, please use a black pen on blank white paper and a high-quality scanner app (or an actual scanner, not just a phone camera).

☞ Some important course policies ☜

- **You may use any source at your disposal**—paper, electronic, or human—but you ***must*** cite ***every*** source that you use, and you ***must*** write everything yourself in your own words. See the academic integrity policies on the course web site for more details.

- **Avoid the Three Deadly Sins!** Any homework or exam solution that breaks any of the following rules will be given an ***automatic zero***, unless the solution is otherwise perfect. Yes, we really mean it. We're not trying to be scary or petty (Honest!), but we do want to break a few common bad habits that seriously impede mastery of the course material.
  - Always give complete solutions, not just examples.
  - Always declare all your variables, in English. In particular, always describe the specific problem your algorithm is supposed to solve.
  - Never use weak induction.

See the course web site for more information.

If you have any questions about these policies,
please don't hesitate to ask in class, in office hours, or on Piazza.

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1. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, either prove that the language is regular (by constructing a DFA or regular expression) or prove that the language is not regular (using fooling sets). Recall that Σ^+ denotes the set of all nonempty strings over Σ.

   (a) $L_{2a} = \{0^n 1^n w \mid w \in \Sigma^* \text{ and } n \geq 0\}$

   (b) $L_{2b} = \{w 0^n w \mid w \in \Sigma^* \text{and } n > 0\}$

   (c) $L_{2c} = \{xwwy \mid w, x, y \in \Sigma^+\}$

   (d) $L_{2d} = \{xwwx \mid w, x \in \Sigma^+\}$

2. Describe the context-free grammar that describes each of the following languages:

   (a) All strings in $\{0, 1\}^*$ whose length is divisible by 5.

   (b) $L_{3b} = \{0^i 1^j 2^{i+j} \mid i, j \geq 0\}$

   (c) $L_{3c} = \{0^i 1^j 2^k \mid i = j \text{ or } j = k\}$

   (d) $L_{3d} = \{w \in \{0, 1\}^* \mid \#(01, w) = \#(10, w)\}$ (function $\#(x, w)$ returns the number of occurrences of a substring x in a string w)

3. An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \Sigma^*$ if **every** possible state that M could be in after reading input x is a state from F. Note, this is in contrast to an ordinary NFA that accepts a string if some state among these possible states is a an accept state. Prove that all-NFAs recognize the class of regular languages.

4. Prove this language is not regular by providing a fooling set. Be sure to include the fooling set you construct is i) infinite and ii) a valid fooling set.

$$L_{P5} = \{w | w \text{ such that } |w| = \lceil k\sqrt{k} \rceil, \text{ for some natural number} k\}$$

Hint: since this one is more difficult, we'll even give you a fooling set that works: try $F = \{0^{m^6} | m \geq 1\}$. We'll also provide a bound that can help: the difference between consecutive strings in the language, $\lceil (k+1)^{1.5} \rceil - \lceil k^{1.5} \rceil$, is bounded above and below as follows

$$1.5\sqrt{k} - 1 \leq \lceil (k+1)^{1.5} \rceil - \lceil k^{1.5} \rceil \leq 1.5\sqrt{k} + 3$$

All that's left is you need to carefully prove that F is a fooling set for L.