This is a review of context-free grammars from the lecture on Tuesday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \rightarrow \epsilon \mid S\,(\,S\,)$$ properly nested parentheses

  Here is a different grammar for the same language:

$$S \rightarrow \epsilon \mid (\,S\,) \mid SS$$ properly nested parentheses

- $\{0^m 1^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of 0s followed by a different number of 1s.

$$
\begin{aligned}
S &\rightarrow A \mid B & \{0^m 1^n\}\, m \neq n \\
A &\rightarrow 0A \mid 0C & \{0^m 1^n\}\, m > n \\
B &\rightarrow B1 \mid C1 & \{0^m 1^n\}\, m < n \\
C &\rightarrow \epsilon \mid 0C1 & \{0^m 1^n\}\, m = n
\end{aligned}
$$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1. $\left\{0^{2n} 1^n \mid n \geq 0\right\}$

2. $\{0^m 1^n \mid m \neq 2n\}$

   [Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string 01.]

3. $\{0, 1\}^* \setminus \left\{0^{2n} 1^n \mid n \geq 0\right\}$

   [Hint: Extend the previous grammar. What is missing?]

The next few problems deal with push-down automata (PDA). The goal of these problems is to simply gain an understanding of PDAs which are the machines needed to recognize a context-free language:

4. What language does the following push-down automata recognize (Hint: This is a non-deterministic automata as most PDAs are)?
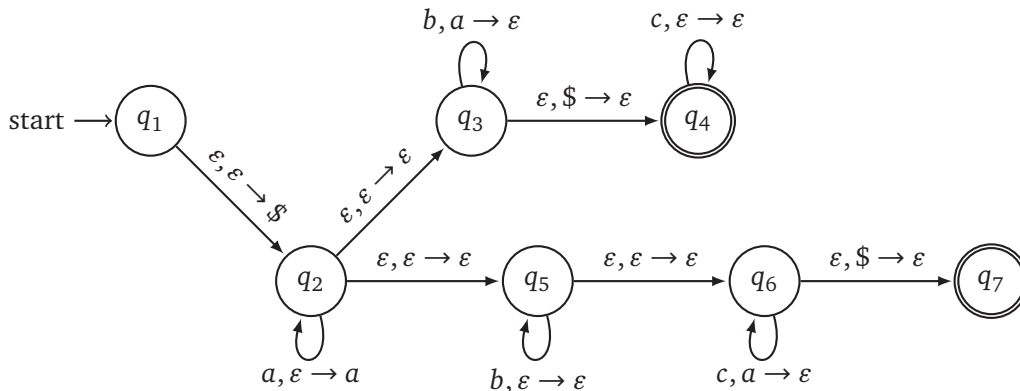
5. Develop the PDA for the language:

$$L = \{w \text{ is a palidrome and } w \in \{\mathbf{0}, \mathbf{1}\}^*\} \tag{1}$$

Work on these later:

4. $\{w \in \{\mathbf{0}, \mathbf{1}\}^*\} \, \#(\mathbf{0}, w) = 2 \cdot \#(\mathbf{1}, w)$ – Binary strings where the number of $\mathbf{0}$s is exactly twice the number of $\mathbf{1}$s.

5. $\{\mathbf{0}, \mathbf{1}\}^* \setminus \{ww\} \, w \in \{\mathbf{0}, \mathbf{1}\}^*$.

   *[Anti-hint: The language $\{ww\} \, w \in \mathbf{0}, \mathbf{1}^*$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]*

6. Convert the following CFG into a PDA:

$$S \to \mathbf{a}B\mathbf{c} \mid \mathbf{ab}$$
$$B \to SB \mid \varepsilon$$