1 Recursive Definitions

Give the recursive definition of the following languages. For both of these you should concisely explain why your solution is correct.

1. A language that contains all strings.
2. A language which holds all the strings containing the substring 000.
3. A language \( L_A \) that contains all palindrome strings using some arbitrary alphabet \( \Sigma \).
4. A language \( L_B \) that does not contain either three 0’s or three 1’s in a row. E.g., \( 001101 \) \( \in L_B \) but \( 10001 \) is not in \( L_B \).

2 Regular Expressions

Give regular expressions for each of the following languages over the alphabet \( \{0, 1\} \).

1. All strings containing the substring 000.
2. All strings not containing the substring 000.
3. All strings in which every run of 0s has length at least 3.
4. All strings in which 1 does not appear after a substring 000.
5. All strings containing at least three 0s.
6. Every string except 000. [Hint: Don’t try to be clever.]
7. All strings \( w \) such that in every prefix of \( w \), the number of 0s and 1s differ by at most 1.

*8. All strings containing at least two 0s and at least one 1.

*9. All strings in which the substring 000 appears an even number of times.
   (For example, 001000 and 0000 are in this language, but 00000 is not.)